# A DESCENDING-THIRDS MODEL OF FUNCTIONAL HARMONY

In many broadly tonal styles—including early music, contemporary popular music, and numerous folk musics—any diatonic triad can progress to virtually any other diatonic triad. In this respect, Western classical music is exceptional: here, a root position V is overwhelmingly likely to progress to a root-position I, whereas it moves to root-position IV only very rarely. Theorists characterize these harmonic regularities in various ways—some use figured bass notation, others Roman numerals; some postulate a preference for certain root motions (e.g. descending over ascending thirds), while others group chords into broader ("functional") categories (see Wason 1984, Lester 1992, Agmon 1995, Meeus 2000, Tymoczko 2003, Quinn 2005). Still others seem to reject the very notion of harmonic regularity, preferring descriptions that unite the harmonic and contrapuntal domains.

Linguists use "corpus studies" to test syntactic theories against sizable databases of English sentences, but music theorists have no comparable analogue. In large part, this is for technological reasons: though the internet contains numerous MIDI files, computers cannot yet translate this information into the familiar language of music theory (e.g. "ii$^6_5$ chord"). To test harmonic theories, we would therefore need a substantial body of hand-made analyses. In my talk I report on what is perhaps the first attempt to create a large, public, machine-readable database of this sort: analyses of the complete Mozart piano sonatas, assembled (and proofread) by more than two dozen professional theorists. (There also exists a much smaller database of 30 Bach chorales, constructed by Craig Sapp.[1]) Following some introductory methodological remarks, the first part of my talk uses this database to propose a new model of functional harmony, one that interacts with recent geometrical models of musical structure (Callender 2004, Tymoczko 2006, Callender, Quinn, and Tymoczko 2008). I conclude by considering the relation between traditional harmonic theory and Schenkerian approaches.

Figure 1 presents a simple thirds-based model of functional harmony, in which chords can move to the right by any number of steps, but can move leftward only along one of the arrows. Figure 2 shows that the model accounts for 95% of the progressions in a selection of Bach chorales and in the complete Mozart piano sonatas, with most of the exceptions falling into a small number of categories: sequences (to be considered momentarily), parallel first-inversion triads, and chromatic chords such as Neapolitans and augmented sixths. Figure 3 shows that the

[1] The data produced by earlier statistically-minded theorists, such as McHose 1947 and Budge 1943 is not publicly available. Sapp's analyses do not attempt to show modulations, and hence are of limited utility.

cycle of thirds plays another role in tonal theory, representing single-step voice-leadings among diatonic triads; for this reason, the circle appears naturally in the orbifold $\mathbb{T}^3/\mathcal{S}_3$, which Tymoczko uses to model three-note chords. Thus the passage in Figure 4 has a double significance: its upper voices are at once a sequence of triads linked by maximally efficient voice-leading, as well as a complete statement of the descending-third chain at the heart of the thirds-based model of functional tonality proposed here.

One attractive explanation for this double function is that common tones and efficient voice-leading together create the effect of harmonic *similarity* (Quinn 2001, Callender, Quinn, and Tymoczko 2008). Figure 5 shows that a root-position F-major triad is very similar to a first inversion D minor triad, since the chords share two notes, with the third separated by just a step. It follows that one chord can replace the other without much disrupting the music's harmonic or contrapuntal fabric: the "substitute" chord will share the bass note and upper third (F and A), differing only by exchanging perfect fifth and consonant sixth (D for C or vice versa). The resemblance between F and $d^6$ is captured implicitly by figured-based notation, which highlights their shared bass note. The thirds-based model uses geometry for a similar purpose: since third-related chords are adjacent on the model, one can replace the other in the rightward progression from tonic to dominant. (Note that this is not always true of progressions that exploit leftward-pointing arrows—thus IV but not $ii^6$ progresses directly to I.) What results is a (largely) root-functional theory that incorporates some insights from the figured-bass tradition.

In normal harmonic contexts, there is no expectation that composers will take particularly short steps along the circle of thirds, since chords are permitted to move arbitrarily far to the right. However, *sequences* often utilize small motions along the descending circle of thirds (Caplin 1998, Harrison 2003, and Ricci 2004). Figure 6 lists all eighteen diatonic sequences whose repeating unit contains at most two chords. In each of the first seven rows, the sequence on the left is considerably more common than that on the right, indicating that descending thirds are indeed preferred. Figure 7 shows the distribution of two-chord sequences in the Mozart piano sonatas, in which the asymmetry between columns is striking. (Figure 8 shows a few examples of the "down a third, up a step" sequence in tonal music—a sequence that has a natural place in this model, but which is only rarely discussed in the literature.) The model thus provides a link between what Fétis (1840/1994) called "harmonic" and "sequential" tonality, suggesting that both exploit the descending circle of thirds in different ways—in harmonic tonality, the circle of thirds organizes motion from tonic to dominant, while in sequential tonality the gravitational pull of these chords is broken, replaced by a preference for short motions along the descending circle.

In the last part of my talk, I consider whether the preceding observations conflict with a Schenkerian approach. I distinguish three ways of understanding the relation between Schenkerian and traditional theory. *Incompatibilists* claim that the harmonic regularities in functional music are only apparently harmonic, and can be explained at a deeper level by contrapuntal principles. *Holists* believe that the very attempt to separate harmony from counterpoint is itself illicit. Finally, *compatibilists* believe that traditional harmonic theory is accurate as far as it goes; on this interpretation Schenkerian analysis adds information to, but does not supplant or conflict with, traditional harmonic analysis. I argue that the success of the thirds-based model strongly suggests (though does not conclusively prove) that compatibilism is the best alternative: the thirds-based model appears to provide an accurate, if approximate, theory of functional harmony, one that is largely independent of contrapuntal considerations; furthermore, there is at present no purely contrapuntal explanation for the fact that the progressions in Figure 9 are rare.

I conclude by observing that this compatibilist *rapprochement* between traditional and Schenkerian theory involves a careful demarcation between their respective domains. I propose that traditional harmonic theory can profitably be understood as the analogue of a linguistic grammar: it provides a comprehensive and principled specification of the harmonic patterns found in tonal music. In my view, it does not necessarily suggest a method of musical *analysis*, any more than the discipline of linguistics mandates a specific approach to poetic interpretation. Nor need it necessarily be understood as making assertions about listeners' psychology: instead, I argue that traditional harmonic theory can be understood as providing a *composers' grammar*—a descriptions of patterns that can be found clearly in scores, presumably because of facts about composers' psychology—to which listeners may or may not be sensitive. Finally, the theory says nothing about whether the principles of functional tonality are conventional or are founded in natural laws. Understood in this way, traditional harmonic theory emerges as a modest but fairly well-confirmed scientific hypothesis. One simple geometrical picture, which can be explained to a first-year theory class in less than an hour, describes the large majority of chord progressions encountered in at least one body of canonical classical music.

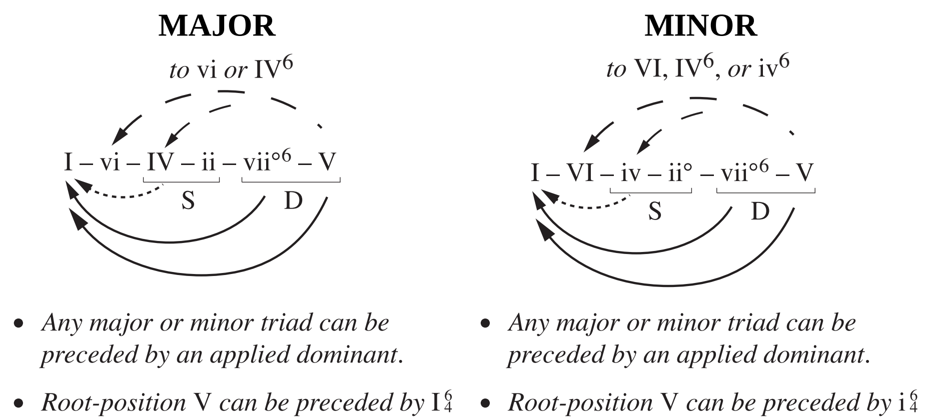



**Figure 1.** A simple model of the allowable chord progressions in functional harmony. Chords can move rightward by any distance, but can move left only along the arrows.

| | Number of progressions | Violations |
|---|---|---|
| Mozart diatonic major | 7955 | 334 (4%) |
| Mozart diatonic minor | 1577 | 90 (5%) |
| Tonicizing progressions | 1154 | 71 (6%) |
| Bach major | 664 | 12 (2%) |

**Figure 2.** Violations of the thirds-based harmonic model in 30 Bach chorales and the complete Mozart sonatas.

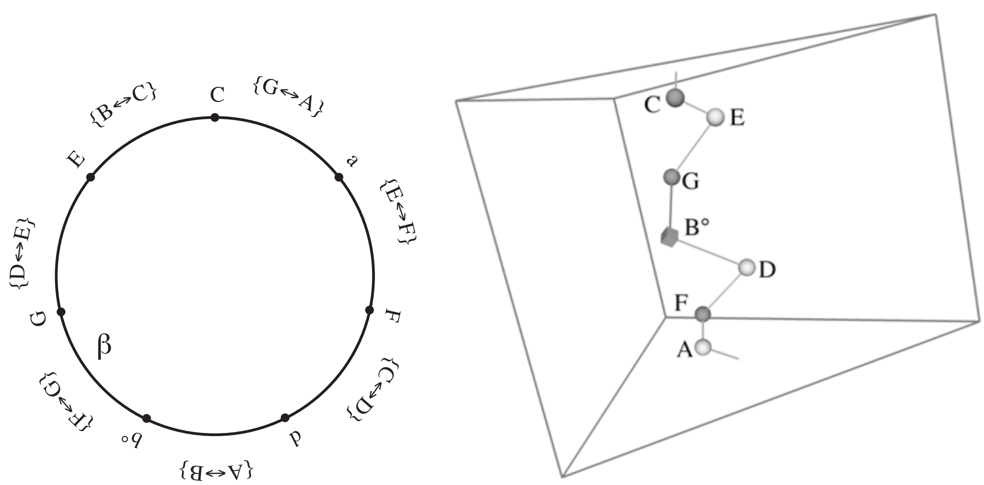



**Figure 3.** The diatonic circle of thirds describes single-note voice leading among diatonic triads, and appears naturally in the three-dimensional space representing three-note chords.

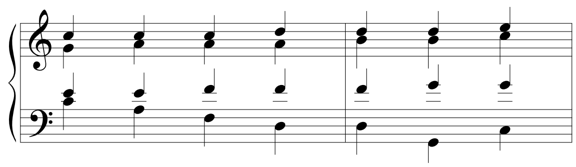
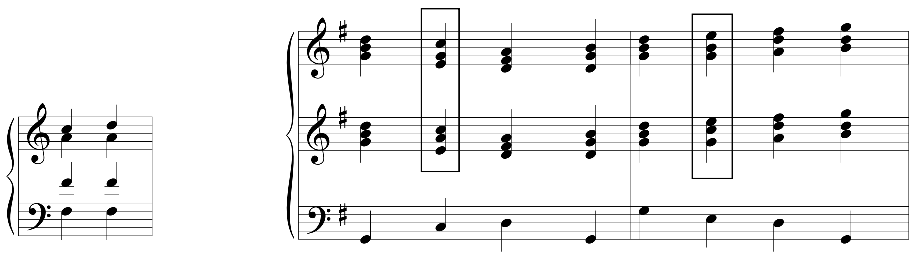


**Figure 4.** Here, the upper voices are connected by single-step voice leading, while the harmonies move along the descending circle of thirds from tonic to dominant.



(*a*) (*b*)

**Figure 5.** (*a*) Third-related triads sound similar, since they share two of their three notes and can be connected by single-step voice leading. (*b*) One can often replace a diatonic chord with a third-related chord, without much disrupting the harmonic or contrapuntal fabric of a passage.

| **Sequence** | | | **Inverted Form** | | |
|---|---|---|---|---|---|
| *a* | *b* | *c* | *a* | *b* | *c* |
| ↓third<br>↓third | –1, –1 | *exists*<br>(C-a-F-d-) | ↑third<br>↑third | +1, +1 | *v. rare*<br>(C-e-G-b°-) |
| ↓third<br>↓fifth | –1, –2 | *v. common*<br>(C-a-d-b°-) | ↑third<br>↑fifth | +1, +2 | *v. rare*<br>(C-e-b°-d-) |
| ↓fifth<br>↓fifth | –2, –2 | *v. common*<br>(C-F-b°-e-) | ↑fifth<br>↑fifth | +2, +2 | *exists*<br>(C-G-d-a-) |
| ↓third<br>↑step | –1, –3 | *exists*<br>(C-a-b°-G-) | ↑third<br>↓step | +1, +3 | *v. rare*<br>(C-e-d-F-) |
| ↑step<br>↓fifth | –3, –2 | *common*<br>(C-$D^7$-G-$A^7$-) | ↓step<br>↑fifth | +3, +2 | *v. rare*<br>(C-b°-F-e-) |
| ↑third,<br>↓fifth | +1, –2 | *common*<br>(C-$E^7$-a-$C^7$-) | ↓third,<br>↑fifth | –1, +2 | *exists*<br>(C-G-e-b°-) |
| ↑fifth,<br>↑step | +2, –3 | *common*<br>(C-G-a-e-) | ↓step,<br>↓fifth | –2, +3 | *exists*<br>(a-$G^7$-C-$B^7$-e-) |
| ↑step,<br>↑third | –3, +1 | *v. rare*<br>(C-d-F-G-) | ↓step,<br>↓third | +3, –1 | *v. rare*<br>(C-b°-G-F-) |
| ↓step<br>↓step | +3, +3 | *exists**<br>(C-d-e-F-) | ↑step<br>↑step | –3, –3 | *exists**<br>(C-d-e-F-) |

**Figure 6.** The eighteen diatonic sequences whose unit of repetition contains at most two chords. The (*a*) columns describe the sequence, the (*b*) columns represent it as a pair of steps on the descending circle of thirds, and the (*c*) columns estimate its frequency in the baroque and classical literature. Sequences featuring descending motion along the circle of thirds, shown in the left column, are more common than their counterparts on the right.

| Sequence | | Inverted Form | |
|---|---|---|---|
| ↓third<br>↓third | 2 | ↑third<br>↑third | 0 |
| ↓third<br>↓fifth | 6 | ↑third<br>↑fifth | 0 |
| ↓fifth<br>↓fifth | 33 | ↑fifth<br>↑fifth | 0 |
| ↓third<br>↑step | 8 | ↑third<br>↓step | 0 |
| ↑step<br>↓fifth | 22 | ↓step<br>↑fifth | 0 |
| ↑third,<br>↓fifth | 1 | ↓third,<br>↑fifth | 0 |
| ↑fifth,<br>↑step | 9 | ↓step,<br>↓fifth | 0 |
| ↑step,<br>↑third | 0 | ↓step,<br>↓third | 0 |

**Figure 7.** Two-unit sequences, of at least six chords in length, in the Mozart piano sonatas.

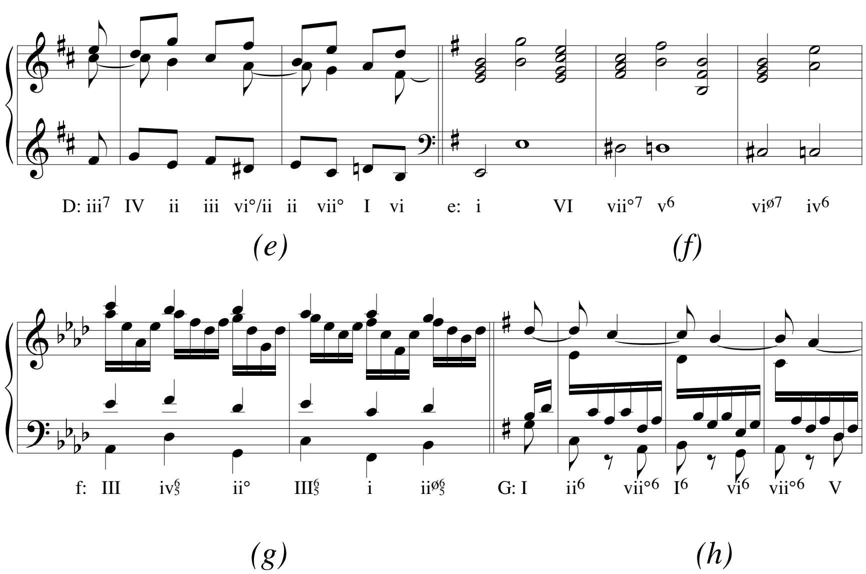
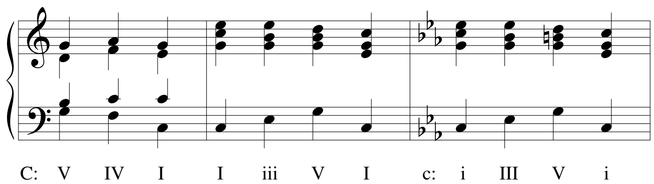



**Figure 8.** The "up a step, down a third" sequence. (*e*) Haydn's D major Piano Sonata no. 56 (Hob. XVI/42) 2, mm. 11-12. (*f*) The opening of the Crucifixus, from Bach's B minor Mass (BWV 232). (*g*) Brahms's F minor Piano Quintet, Op. 34, I, mm. 8-9. (h) Bach's G major Fugue, Book II of the *Well-Tempered Clavier*, mm. 66-69. Not all theorists read these sequences as exemplifying the "down a third, up a step" pattern. The descending-thirds model suggests that we might want to learn to hear the similarities between these various progressions.




| | |
|---|---|
| V-vi | 103 |
| I-I$^6$-V | 56 |
| V-IV$^6$ | 48 |
| i-i$^6$-V | 10 |
| V-IV-I | 2* |
| I-iii-V | 0 |
| i-III-V | 0 |

(*a*) (*b*)

**Figure 9.** (*a*) Three progressions that are contrapuntally unobjectionable, but which rarely appear in the Mozart sonatas: a root-position V-IV, a major-key I-iii-V-I, and the analogous progression in minor. (*b*) Chord progressions in Mozart's sonatas. The last three progressions in the list are indeed quite rare, as asserted by the current model. The starred progressions all occur across phrase boundaries (K. 311, movement 3, mm. 71–72.)

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