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ROOT MOTION, FUNCTION, SCALE-DEGREE:

a grammar for elementary tonal harmony

The paper considers three theories that have been used to explain tonal harmony: root-motion theories, which emphasize the intervallic distance between successive chord-roots; scale-degree theories, which assert that the triads on each scale degree tend to move in characteristic ways; and function theories, which group chords into larger ("functional") categories. Instead of considering in detail actual views proposed by historical figures such as Rameau, Weber, and Riemann, I shall indulge in what the logical positivists used to call "rational reconstruction." That is, I will construct simple and testable theories loosely based on the more complex views of these historical figures. I will then evaluate those theories using data gleaned from the statistical analysis of actual tonal music.

The goal of this exercise is to determine whether any of the three theories can produce a simple "grammar" of elementary tonal harmony. Tonal music is characterized by the fact that certain progressions (such as I-IV-V-I) are standard and common, while others (such as I-V-IV-I) are nonstandard and rare. A "grammar," as I am using the term, is a simple set of principles that generates all and only the standard tonal chord progressions. I shall describe these chord progressions as "syntactic," and the rare, nonstandard progressions as "nonsyntactic."[1] This distinction should not be taken to imply that nonsyntactic progressions never appear in works of tonal music: some great

[1] Intuitions about the grammaticality of chord-sequences and natural language sentences are importantly different, not least in that the semantics of natural language reinforces our intuitions about syntax. Nongrammatical sentences of natural language often lack a clear meaning. This helps to create very strong intuitions that these sentences are (somehow) "wrong," or "defective." Chord-sequences, even well-formed ones, do not have meaning. This means that their grammaticality is more closely related to their statistical prevalence: even a "nonsyntactic" tonal progression like I-V-IV-I sounds less "wrong" than "unusual" (or "nonstylistic"). Nevertheless, there is an extensive pedagogical and theoretical tradition which attempts to provide rules and principles for forming "acceptable" chord-progressions. It seems reasonable to use the word "syntactic" in connection with this enterprise.

tonal music contains nonsyntactic chord progressions, just as some great literature contains nongrammatical sentences. Nevertheless, we do have a good intuitive grasp of the difference between standard and nonstandard progressions. My question is whether any of the three theories considered provide a clear set of principles that accurately systematizes our intuitions about tonal syntax.

The term "tonal music" describes a vast range of musical styles from Monteverdi to Coltrane. It is clearly hopeless to attempt to provide a single set of principles that describes all of this music equally well. Following a long pedagogical tradition, I will therefore be using Bach's chorale harmonizations as exemplars of "elementary diatonic harmony." I will also make a number of additional, simplifying approximations. First, I will confine myself exclusively to major-mode harmony. Second, I will, where possible, discard chord-inversions. This is because tonal chord progressions can typically appear over multiple bass lines. (Exceptions to this rule will be noted below.) Third, I will disregard the difference between triads and seventh chords. This is because there are very few situations in which a seventh chord is required to make a progression syntactic; in general, triads can be freely used in places where seventh chords are appropriate.[2] Fourth, I will for the most part consider only phrases that begin and end with tonic triads. Tonal phrases occasionally begin with nontonic chords, and frequently end with half-cadences on V. However, these phrases are often felt to be unusual or incomplete—testifying to a background expectation that tonal phrases should end with the tonic. Finally, I will be considering only diatonic chord progressions. It is true that Bach's major-mode chorales frequently involve modulations, secondary dominants, and the use of other chords foreign to the tonic scale. But these chromatic harmonies can often be understood to embellish a more fundamental, purely diatonic substrate.

Historians may well feel that I am drawing overly sharp distinctions between root-motion, scale-degree, and functional theories. Certainly, many theorists have drawn freely on all three traditions. (Rameau in particular is an important progenitor of all of the theories considered in this paper.) In treating these three theories in isolation, it may

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[2] There are some exceptions to this rule. Bach avoided using the root-position leading-tone triad, though he used the leading-tone seventh chord in root position. Since I am disregarding inversions, this does not create problems for my view.

therefore seem that I am constructing straw-men, creating implausibly rigid theories that no actual human being has ever held—and that cannot describe any actual music. It bears repeating, therefore, that my goal here is not a historical one. It is, rather, to see how well we can explain the most elementary features of tonal harmony on the basis of a few simple principles. In doing so, we will hopefully come to appreciate how these various principles can be combined.

1. Root-motion theories.

a) Theoretical perspectives.

Root-motion theories descend from Rameau (1722) and emphasize the relations between successive chords rather than the chords themselves. A pure root-motion theory asserts that syntactic tonal progressions can be characterized *solely* in terms of the type of root motion found between successive harmonies. Good tonal progressions feature a restricted set of root motions, such as motion by descending fifth or descending third; bad tonal progressions feature "atypical" motion, such as root motion by descending second. Figures such as Rameau, Schoenberg (1969), Sadai (1980), and Meeus (2000), have all explored root-motion theories. In most cases, these writers have supplemented their theories with additional considerations foreign to the root-motion perspective. Meeus, however, comes close to articulating the sort of pure root-motion theory that we shall be considering here.

A pure root-motion theory involves two principles. The first might be called *the principle of scale-degree symmetry*. This principle asserts that *all* diatonic harmonies participate equally in the same set of allowable root motions. It is just this principle that distinguishes root-motion theories—which focus on the intervallic distance between successive harmonies—from more conventional views, in which individual harmonies are the chief units of analysis. As we shall see, this is also the most problematic aspect of root-motion theories. It is what led Rameau to supplement his root-oriented principles with arguments about the distinctive voice-leading of the V^7-I progression. In this way, he was able to elevate the V-I progression above the other descending-fifth progressions in the diatonic scale.

The second principle is *the principle of root-motion asymmetry,* which asserts that certain types of root motion are preferable to others. For example: in tonal phrases, descending-fifth root motion is common, while ascending-fifth root motion is relatively rare. (The strongest forms of this principle absolutely forbid root motion by certain intervals, as Rameau did with descending seconds.) Meeus and other root-motion theorists take these asymmetries to characterize the difference between modal and tonal styles.

What is particularly attractive about root-motion theories is the way they promise to provide an explanation of functional tendencies. These tendencies are often thought to be explanatorily basic: for many theorists, it is just a brute fact that the V chord tends to proceed downward by fifth to the I chord, one that cannot be explained in terms of any more fundamental musical principles. Likewise, it is just a fact that a "subdominant" IV chord tends to proceed up by step to the V chord. Root-motion theories, by contrast, promise to provide a deeper level of explanation, one in which each tonal chord's individual propensities can be explained in terms of a small, shared set of allowable root motions.

To see how this might work, let us briefly consider the details of Meeus's theory. Meeus (2000) divides tonal chord progressions into "dominant" and "subdominant" types. For Meeus, root motion by fifth is primary: descending-fifth motion represents the prototypical "dominant" progression, while ascending-fifth motion is prototypically "subdominant." Meeus additionally allows two classes of "substitute" progression: root-progression by third can "substitute" for a fifth-progression in the same direction; and root-progression by step can "substitute" for a fifth-progression in the opposite direction. These categories are summarized in Example 1, which has been reprinted from Meeus (2000). Meeus does not explicitly say why third-progressions can substitute for fifth progressions, but his explanation of the second sort of substitution follows Rameau.[3] For Meeus, ascending-step progressions such as IV-V, represent an elision of an intermediate harmony which is a third below the first chord and a fifth above the second. Thus a IV-V

[3] Schoenberg classifies descending-fifth and descending-third progressions together because in these progressions the root note of the first chord is preserved in the second. Meeus presumably has something similar in mind.

progression on the surface of a piece of music "stands for" a more fundamental IV-ii-V progression that does not appear. The insertion of this intermediate harmony allows the seemingly anomalous IV-V progression to be explained as a series of two "dominant" progressions, one a "substitute" descending-third progression, the other descending by fifth.

Consider now Example 2, which arranges the seven major-scale triads in descending third sequence. Meeus's three types of "dominant" progression can be explained by three types of rightward motion along the graph of Example 2. Descending-fifth progressions represent motion two steps to the right. Descending third progressions represent motion a single step to the right. Ascending seconds represent motion three steps to the right, eliding a descending third progression (one step to the right) with a descending fifth progression (two more steps to the right). Meeus's view is that these three types of rightward motion together constitute the allowable moves in any "well-formed" tonal progression.

This theory, as it stands, is problematic. The first difficulty is that normal tonal phrases tend to begin and end with the tonic chord. A pure root-motion theory has difficulty accounting for this fact, for it requires privileging the I chord relative to the other diatonic harmonies. This runs counter to the principle of scale-degree symmetry. Indeed the very essence of root-motion theories is to argue that root motion, and not an abstract hierarchy of chords, determines the syntactic tonal chord progressions. Yet it seems that we must assert such a chordal hierarchy if we are to explain why tonal progressions do not commonly begin and end with nontonic chords. This represents a significant philosophical concession on the part of root-motion theorists. Let us ignore its implications for the moment, however, and simply add an additional postulate to Meeus's system, requiring that syntactic progressions begin and end with the I chord.

The second problem has to do with the iii chord, which has been bracketed in Example 2. Meeus's root-motion theory predicts that progressions such as V-iii-I, ii-iii-I, and vii°-iii-I, should be common. Indeed, from a pure root-motion perspective, such progressions are no more objectionable than progressions such as ii-V-I and vi-IV-V-I. But actual tonal music does not bear this out. Mediant-tonic progressions are extremely rare in the music of the eighteenth and early nineteenth centuries.[4] (They are slightly less rare, though by no means common, in the later nineteenth century.) Again, it seems that we need to extend Meeus's theory by attributing to iii a special status based on its position in an abstract tonal hierarchy. I propose that we eliminate it from consideration, forbidding any progressions that involve the iii chord on Example 2. This amounts to asserting that the iii chord is not a part of basic diatonic harmonic syntax.[5]

We can now return to Example 2, and consider all the chord progressions that a) begin and end with the tonic triad; b) involve only motion by one, two, or three steps to the right; and c) do not involve the iii chord. Considering first only those progressions that involve a single rightward pass through the graph, we find 20 progressions. They are listed in Example 3. Note that we can generate an infinite number of additional progressions by allowing the V chord to move three steps to the right, past the I chord, to the vi chord. (This "wrapping around" from the right side of the graph to the left represents the traditional "deceptive progression.") We will discount this possibility for the moment.

It can be readily seen that all the progressions in Example 3 are syntactic. More interestingly, all of them can be interpreted functionally as involving T-S-D-T (tonic-subdominant-dominant-tonic) progressions. (In half of the progressions, the subdominant chord is preceded by vi, which I have here described as a "pre-subdominant" chord, abbreviated PS.) Perhaps most surprisingly, Example 3 is substantially complete. Indeed, we can specify the progressions on that list by the following equivalent, but explicitly functional, principles:

1. Chords are categorized in terms of functional groups.
    a. the I chord is the "tonic."
    b. the V and vii° chords are "dominant" chords.

[4] The augmented mediant triad occasionally seems to function as a dominant chord in Bach's minor-mode music. However, mediant-tonic progressions are very rare in major. Furthermore, many cases in which mediants appear to function as dominant chords—particularly the first-inversion iii chord in major—are better explained as embellishments of V chords (V^{13} or V "add 6").

[5] Note that the iii chord gets counted, even though the chord itself cannot be used. For example motion from V to I involves moving two steps to the right, even though the iii chord cannot itself participate in syntactic chord progressions.

c. the ii and IV chords are “sub-dominant” chords.
d. the vi chord is a “pre-subdominant” chord.

2. Syntactic progressions move from tonic to subdominant to dominant to tonic.
   a. the first subdominant chord in a T-S-D-T sequence may be preceded by a pre-subdominant chord, though this is not required.
   b. It is allowable to move between functionally identical chords only when the root of the first chord lies a third above the root of the second.

These principles capture, to a reasonable first approximation, an important set of tonally-functional progressions, namely the T-S-D-T progressions.[6] Such sequences are arguably the most prototypical tonal progressions, as they involve the three main tonal functions all behaving in the most typical manner. Thus it even the more remarkable that we have generated all the progressions meeting these criteria without any overt reference to the notion of chord function. Instead, we have derived a notion of tonal function from root-motion considerations. It is true that we have asserted that the I and iii chords have a special status. But beyond that, we have relied on root-motion constraints to generate our functional categories.[7]

The significance of all this is, I believe, a matter that merits further investigation. On the one hand, it may be that in deriving functional progressions from root-motion considerations, we have engaged in a piece of merely formalistic manipulation, devoid of real musical significance. (Particularly suspicious here are the non-root-motion principles by which we have increased the significance of the I chord, and demoted that of the iii chord.) On the other hand, the root-motion principles embodied in Meeus's (modified) theory may indicate a reason for the tonal system's longevity: it is perhaps the preference for “dominant” progressions that explains why T-S-D-T progressions are felt

[6] My functional categories are more restrictive than Riemann's: I consider ii and IV to be the only subdominant chords, and V and vii to be the only dominant chords. For more on this, see Section 2(b), below.
[7] We can expand the progressions on this list by allowing progressions that “wrap around” the graph of Example 2. This is equivalent to adding the following functional principle to 1-2, above:
3*. Dominant chords can also progress to vi as part of a “deceptive” progression.

to be particularly satisfactory. Furthermore, Meeus's theory suggests a plausible mechanism by which the functional categories "subdominant" and "dominant" could have arisen. Meeus himself has proposed that functional tonality arose as composers gradually began to favor "dominant" progressions over "subdominant" progressions. If historians could document this process, it would represent a substantial step forward in the explanation of the origin of tonal harmony. In the next section, I will consider evidence that bears on this issue.

b) Empirical data

Let us informally test Meeus's hypothesis that tonal music involves a preference for "dominant" chord progressions. Example 4(a) presents the results of a computational survey of chord progressions in the Bach chorales. This table was generated from MIDI files of the 186 chorales published by Kirnberger and C.P.E. Bach (BWV nos. 253-438). The analysis that produced this table was extremely unsophisticated: the computer simply looked for successive tertian sonorities (both triads and seventh-chords), and measured the interval between their roots. The computer was unable to recognize passing or other nonharmonic tones, or even to know whether a chord progression crossed phrase-boundaries. Thus a great number of "legitimate" chord progressions, perhaps even the majority of the progressions to be found in the chorales, were ignored. More than a few "spurious" progressions, which would not be considered genuine by a human analyst, were doubtless included. Nevertheless, despite these limitations, the data in Example 4(a) provide a very approximate view of the root-motion asymmetry in Bach's chorales. Example 4(b), by way of contrast, shows the results of a similar survey of a random collection of 17 Palestrina compositions.[8]

Comparison of Examples 4(a) and 4(b) provides limited support for Meeus's theory. There is, as expected, more root-motion asymmetry in Bach's (tonal) chorales than in Palestrina's (modal) mass movements. However, the difference is less dramatic than one might have expected. This is due to two factors: first, there is already a

[8] The pieces were downloaded from the website www.classicalarchives.com.

noticeable asymmetry in Palestrina's modal music.[9] Second, Bach's music involves a higher-than-expected proportion of "subdominant" progressions. Meeus (2000) hypothesizes that fully 90% of the progressions in a typical tonal piece are of the "dominant" type. Example 4(a) suggests that the true percentage is closer to 75%.

Example 5 attempts to explore this issue by way of a more sophisticated analysis of 30 major-mode Bach chorales. These chorales, along with a Roman-numeral analysis of their harmonies, were translated into the Humdrum notation format by Craig Sapp. (The Appendix lists the specific chorales used.) I rechecked, and significantly revised, Sapp's analyses. I then programmed a computer to search the 30 chorales for all the chord progressions that a) began and ended with a tonic chord; and b) involved only unaltered diatonic harmonies. Example 5 lists the 169 resulting progressions, categorized by functional type. The first column of the example lists the actual chords involved. The second analyzes the progression as a series of "dominant" and "subdominant" root motions in Meeus's sense. The third column lists the number of chord progressions of that type found in the 30 chorales.[10]

The results reveal both the strengths and weaknesses of a root-motion approach. On the positive side, the modified root-progression theory we have been considering accurately captures all of the chord progressions belonging to the T-S-D-T functional category, and a majority of the progressions in which vi functions as a pre-subdominant chord (category 4[a] on Example 5). It is also noteworthy that a large number of the possible dominant progressions appear in Example 5. Example 6 lists the five dominant progressions, out of a possible 21, that do *not* appear. It can be seen that all but one of these progressions (vii°–V) involve the iii chord. This is in keeping the view, proposed earlier, that the mediant chord has an anomalous role within the tonal system. By contrast, less than half of the possible subdominant progressions appear in Example 5,

[9] This phenomenon is beyond the scope of this paper. However, the data in Example 4(b) do cast doubt on the simplistic picture of modal music as involving no preference at all for "dominant" over "subdominant" progressions.

[10] Note that throughout Example 5, I have for the most part ignored chord-inversion, and have treated triads and sevenths as equivalent. I have also discounted cadential I^{6}_{4} chords for the purposes of identifying "subdominant" and "dominant" progressions. Here I am following recent theorists in treating these chords as functionally anomalous—perhaps as being the products of voice-leading, rather than as functional harmonies in their own right (see Aldwell and Schachter 2002).

and these are, as Example 7 shows, strongly asymmetrical as to type. Indeed, fully 87% of Example 7's subdominant progressions be accounted for by just three chord progressions: I-V, IV-I, and V-IV^6. The relative scarcity of subdominant progressions, both in terms of absolute numbers, and in terms of the types of chord progressions involved, suggests that there is something right about Meeus's theory. "Dominant progressions" are much more typical of tonal music than "subdominant progressions." They can, as Schoenberg writes, be used more or less "without restriction."

Nevertheless, Example 5 does pose two serious problems for a pure root-motion view of tonality. The first is that subdominant progressions tend to violate the principle of scale-degree symmetry. The second is that these same progressions seem to violate the much deeper principle of root-functionality. I shall briefly discuss each difficulty in turn.

1. Subdominant progressions and scale-degree symmetry. Meeus proposes that a well-formed tonal phrase should consist of "dominant progressions exclusively." Yet the two most common chord progressions in Example 5 both violate this rule. I-V-I and I-IV-I both involve subdominant root motion by ascending fifth. Other common progressions involve similarly forbidden types of root motion: V-IV^6, which appears 10 times in Example 5, and vi-V, which appears three times, both involve root motion by descending second. vi-I^6, which appears four times, involves root motion by ascending third.

Schoenberg and Meeus both try to provide rules that account for such progressions solely in terms of root-motion patterns. Schoenberg writes:

> Descending progressions [i.e. progressions in which roots *ascend* by third or fifth, which Meeus calls "subdominant"], while sometimes appearing as a mere interchange (I-V-V-I, I-IV-IV-I), are better used in combinations of three chords which, like **I**-V-**VI** or **I**-III-**VI**, result in a strong progression.[11]

Meeus's view is that while tonal progressions may sometimes involve "subdominant"

[11] Schoenberg 1969, 8.

progressions, these are not normally found in direct succession.[12] This suggests a root-motion principle according to which isolated subdominant progressions can be freely inserted into chains of dominant progressions.

Neither of these proposals can account for the data in Example 5. The fundamental problem is that the subdominant progressions in Example 5 strongly violate the principle of scale degree symmetry. For example: though some ascending-fifth progressions are very common (e.g. I-V, IV-I), others do not appear at all (e.g. V-ii, vii°-IV). Likewise, while progressions like vi-V and vi-I^6 are relatively common, other progressions involving similar root motion—for instance, ii-I, and I-iii^6—are not. This means that pure root-motion theories will have serious difficulties accounting for the role of subdominant root-progressions in elementary tonal harmony. For these progressions violate the cardinal principle of root-motion theories, namely scale-degree symmetry.

Note that, in contrast to the subdominant progressions, the dominant progressions do by and large tend to obey the principle of scale-degree symmetry. While it is true that some dominant progressions (such as V-I) appear more than others, it is also true that, with the exception of those progressions listed in Example 6, the dominant progressions are all fairly common. This is in keeping with the root-motion principle that diatonic triads can freely move by way of descending fifths and thirds, or by ascending second. Aside from the anomalous mediant triad, the sole exception to this rule concerns the vii° chord, which tends to ascend by step rather than descending by third or fifth.

2. Inversion-specific subdominant progressions. A second and more interesting difficulty is that some subdominant progressions typically involve specific chords *in specific inversions*. For example: a root-position IV chord does not typically occur after a root-position dominant triad, though the progression V-IV^6 is quite common. This fact represents a challenge not just to root-motion theories, but to the very notion of root-functionality—that is, to the very notion that one can determine the syntactic chord progressions solely by considering the root of each chord.[13] The presence of inversion-

[12] This assertion is inconsistent with his assertion that "well-formed" progressions consist entirely of dominant progressions.

[13] Schoenberg (1969, p. 6) writes: "The structural meaning of a harmony depends exclusively on the degree of the scale. The appearance of the third, fifth, or seventh in the bass serves only for greater variety in the 'second melody.' Structural functions are asserted by *root progression*" (Schoenberg's italics).

specific chord progressions reminds us that the almost universally accepted principle of root-functionality is in fact only an approximation.

A good number of these inversion-specific progressions can be attributed to the intersubstitutability of IV^6 and vi.[14] The anomalous vi in a vi-I^6 progression can be understood as substituting for the IV^6 chord in the more typical (though still "subdominant") IV^6-I^6 progression. Likewise, one can interpret the atypical V-IV^6 progression as involving the substitution of IV^6 for vi. The fact that these chords are similar is not altogether surprising, since they share two common pitches and the same bass note. It is as if vi and IV^6 were two versions of the same chord, one having a perfect fifth above the bass, the other a minor sixth. Putting the point in this way suggests that the principle of *bass-functionality*, rather than root-functionality, may be needed to explain the resemblance between IV^6 and vi. Clearly, it is difficult for root-motion theories to account for this fact. Since they are strongly committed to the principle of root-functionality, these theories must treat vi and IV^6 as fundamentally different harmonies.

2. Scale degree and function theories

a) Scale-degree theories

Scale-degree theories descend from Vogler (1776) and Weber (1817-21), and begin with the postulate that diatonic triads on different scale degrees each move in their own characteristic ways. This postulate underwrites the familiar practice of Roman-numeral analysis. By identifying each chord's root, and assigning it a scale-degree number, the scale-degree theorist purports to sort diatonic chords into functional categories.[15] Thus scale-degree theorists cut the Gordian knot that besets root-motion theorists: abandoning the principle of scale-degree symmetry, they allow that different diatonic triads may participate in fundamentally different kinds of motion.

Scale degree theories are often represented by a map showing the allowable transitions from chord to chord. (Example 8 reprints the map from Stefan Kostka and

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[14] This intersubstitutability is highlighted in Aldwell and Schachter 2002.
[15] I am here using the term "function" in a broad sense. The point is that chords sharing the same root tend to behave in similar ways.

Dorothy Payne's harmony textbook.[16]) Scale-degree theories can also be represented by what are called *first-order Markov models*. A first-order Markov model consists of a set of numbers representing the probability of transitions from one "state" of a system to another. In the case of elementary diatonic harmony, the "states" of the system represent individual chords. Transition probabilities represent the likelihood of a progression from a given chord to any other. Thus a simple scale-degree theory of elementary diatonic harmony can be expressed as a 7 x 7 matrix representing the probability that any diatonic chord will move to any other.[17]

Example 9 presents such a matrix, generated by statistical analysis of Bach chorales. To produce this table, I surveyed all the 2-chord diatonic progressions in the 30 chorales analyzed by Sapp. A total of 956 progressions were found.[18] This table is meant to be read from left to right: thus, moving across the first row of Example 9, we see that 23% of the I chords (73 out of a total of 315 progressions) "move" to another I chord; 11% of the progressions (36 out of 315) move to a ii chord; 0% move to a iii; 23% move to a IV; and so on. Perusing the table shows that the different chords do indeed tend to participate in fundamentally different sorts of root motion. Fully 81% of the vii° chords proceed up by step to a I chord, whereas only 11% of the I chords move up by step. Likewise, almost a third (31%) of the I chords move up by fifth, compared to a mere 1% of the V chords. These results provide yet another reason for rejecting the principle of scale-degree symmetry, and with it, pure root-motion accounts of diatonic harmony.

Example 10 explores a modified version of the matrix given in Example 9. Here I have altered the numbers in Example 9, in order to produce the closest approximation to the chord progressions listed in Example 5. The actual probability values that I used are given in Example 10(a); Example 10(b) lists a random set of 169 chord progressions

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[16] Kostka and Payne 2000.

[17] A Markov model is superior to a harmonic map in that it can show the relative frequency of chord progressions. Thus, while a map might indicate that one may progress from ii to V and vii, the Markov model also shows how likely these transitions are.

[18] This number is much higher than the number of progressions found in Example 5. In order to obtain the largest possible number of progressions, I permitted phrases containing nondiatonic triads. One should threfore treat these numbers as approximate: Sapp analyzed most of the non-diatonic chords in these chorales in terms of the tonic key of the chorale, rather than the local key of the phrase. Thus a I-IV progression in a phrase that modulated to the dominant would be described by Sapp as a V-I progression, since IV in the (local) dominant key is I in the (global) tonic. My analysis here does not correct for this fact.

produced by the model. Comparison of Example 10(b) with Example 5 shows that the first-order Markov model does an excellent job of approximating the progressions found in Bach's chorales. Almost all of the progressions generated by the model are plausible, syntactic tonal progressions. Furthermore, the scale-degree model generates a much greater variety of syntactic progressions than the pure root-motion model considered earlier in Example 3. Finally, the model does a reasonably good job of capturing the relative preponderance of the various types of progression found in Bach's music. In particular, this scale-degree model accurately represents the high proportion of I-V-I and I-IV-I progressions in the chorales.

Nevertheless, a few differences between Bach's practice and the output of the model call for comment.

a. Repetitive progressions. Certain progressions produced by the model are highly repetitive, and seem unlikely to have been written by Bach. For example, the progression I-vi-V-vi-V-I, involves a rather unstylistic oscillation between vi and V. In the progression I-ii-V-vi-IV-V-vi-V-I, the first V-vi progression weakens the effect of the second, spoiling its surprising, "deceptive" character. The problem here is, clearly, that the Markov model has no memory. The probability that a V chord will progress to a vi chord is always the same for every V chord, no matter what comes before it. Such difficulties are endemic to first-order Markov models and can be ameliorated only by providing the system with a more sophisticated memory of past events.[19]

b. IV-I progressions. The model produced two progressions that do not appear in Bach's chorales: I-vi-IV-I and I-V-vi-IV-I. While it is conceivable that Bach could have written such progressions, there is something slightly odd about them: IV-I progressions tend to occur as part of a three-chord I-IV-I sequence; furthermore, such sequences are more likely to occur near the beginning of a phrase (or as a separate, coda-like conclusion to a phrase), than as the normal conclusion of an extended chord progression. This is again a memory issue. The first order Markov model has no way of distinguishing

[19] These problems also beset simple "maps" such as that proposed by Kostka and Payne.

between the typical progression I-IV-I-V-I and the rather more atypical I-V-I-ii-V-vi-IV-I.[20]

c. Non-root-functional progressions. The Markov-model, like the earlier root-motion model, does not reproduce inversion-specific progressions such as vi-I^6 or V-IV^6. This problem is easily correctible. All that is needed is to add new states to the model that represent the I^6 and IV^6 chords. (These states would be very similar to those which represented the root-positions of the same chords; their main function would be to permit progressions like vi-I^6 while ruling out progressions like vi-I.) I have chosen not to do so for the sake of simplicity. Yet it is perhaps an advantage of the scale-degree model that it can easily account for such progressions. By contrast, it is harder to see how one might alter a root-motion theory to account for the existence of inversion-specific chord progressions.

d. Tonal idioms. Tonal music features a number of characteristic medium-length chord sequences such as V-IV^6-V^6 and I-V-vi-iii-IV-I-IV-V. These could be considered "idioms" of the tonal language, in that they are both grammatically irregular and statistically frequent. A pure scale-degree theory cannot account for these progressions. Instead, they need to be added to the model individually, as exceptions that nevertheless typify the style.

Despite these limitations, however, the simple first-order Markov model does a surprisingly good job of approximating the progressions of elementary diatonic harmony. In particular, it does a much better job than the pure root-motion perspective considered in the previous section. But this should not be taken to mean that the root-motion view has been completely superseded. For the scale-degree theory we have been considering *incorporates* some of the principal observations of the previous section. Surveying the matrices in Examples 9 and 10(a), we can see that they themselves validate two of Meeus's claims: "dominant progressions" are indeed more frequent than "subdominant" progressions; and "subdominant" progressions are confined to a smaller set of progression-types. Indeed, it is easy to see that the matrices in Example 9 and 10(a) will generate asymmetrical root-motion statistics of the sort we found earlier (Example 4[a]).

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[20] A similar problem would confront the theorist who tried to incorporate the cadential six-four chord into the model.

By contrast, one cannot generate these matrices themselves from Meeus's pure root-motion principles. In this sense the scale-degree theory is richer than the root-motion view.

Example 11 provides another perspective on the relationship between scale-degree and root-motion theories. Here I have summarized Example 9, identifying the extent to which chords on each scale degree tend to participate in "dominant" and "subdominant" progressions in Meeus's sense. Thus, the first line of Example 11(a) shows that 94% of the two-chord progressions beginning with V are "dominant" progressions in Meeus's sense, while only 6% are "subdominant" progressions. (For the purposes of this table, I have discounted chord-repetitions, which Example 9 shows as root motions from a chord to itself.) We see that there is a striking difference in the degree to which each chord participates in dominant progressions. While the V and the vii° chord move almost exclusively by way of "dominant" progressions, the I chord participates in an almost even balance of "dominant" and "subdominant" root motion.

Example 11(b) shows that root-motion asymmetry in general increases as one moves down the cycle of thirds from I to V. Comparing Example 11(b) to Example 2, we see that in ordering the primary diatonic triads with respect to their tendency to move asymmetrically, we obtain almost the same descending-thirds ordering we used to generate Example 2. Only the iii chord, which in Example 11(b) occurs between IV and ii, disturbs the parallel. (I have placed the chord on its own line in Example 11[b], to heighten the visual relationship between Examples 2 and 11[b].) The resemblance between Examples 2 and 11(b) suggests two thoughts. First, Meeus's contrast between modal and diatonic progressions is actually a very apt description of the difference *between chord-tendencies within the diatonic system*. Recall that Meeus postulated that modal music is characterized by a relative indifference between "dominant" and "subdominant" progressions, while tonal music is characterized by a strong preference for "dominant" root-progressions. Example 11(b) shows that within Bach's tonal language, the I chord moves more or less indifferently by way of dominant and subdominant progressions, while the V and vii° chords are strongly biased toward "dominant" progressions. Thus we could say that chord-motion beginning with I is "modal" in Meeus's sense, while chord-motion beginning with V, vii°, and, to a lesser extent, ii, is "tonal." It is therefore an oversimplification to suggest that tonal harmony *in general* is biased toward "dominant" progressions. Rather, the bias belongs to a limited set of chords within the diatonic universe.

The second thought suggested by Example 11(b) is that Meeus's speculative genealogy of the origins of the tonal system has become much more problematic. Recall that on Meeus's account, the tonal system arose as the result of an increasing preference for "dominant" root-progressions. Example 11(b) suggests that by the time Bach developed his harmonic language, a second process must also have occurred: namely, the loosening of the preference for "dominant" progressions in the case of the tonic and submediant harmonies. I find this two-stage hypothesis somewhat implausible. It seems much simpler to propose that the tonal system arose as the result of an increasing awareness of the V and vii° chords as having a distinctive tendency to progress to I. Recall, in this connection, that Example 9 shows that V and vii° chords both tend to progress by way of *different* "dominant" progressions: the V chord usually moves *down by fifth* to I, whereas the vii° chord tends to move *up by step* to I. What is common is not the type of root motion involved, but rather the fact that both chords tend to move to I. All of this accords much better with the scale-degree rather than the root-motion perspective.

b) Function theories

Function theories descend from Riemann (1893). These theories, as Agmon (1995) emphasizes, have two components. The first groups chords together into categories. For Riemann, V, vii°, and iii together comprise the "dominant" chords; IV, ii, and vi comprise the "subdominant" chords; and I, iii, and vi comprise the "tonic" chords. (Note that iii and vi each belong to two categories.) The second component of a function theory postulates an allowable set of motions between functional categories—usually, motion from tonic to subdominant to dominant and back to tonic. It is a characteristic of many function theories that the categorization of chords and the identification of normative patterns of chord motion proceed by way of different principles. Thus

Riemann categorized chords by way of common-tone-preserving operations such as "relative" and "leading-tone exchange." Identification of normative patterns of functional progression occurs independently.

There are two different ways to understand the notion of chordal "functions." The first, and more common, posits functions as psychological realities, asserting that we hear chords in single functional category as having perceptible similarities. Thus, on this account, the progressions ii-iii-I and IV-V-I are experienced as being psychologically similar, since both involve motion between functionally identical chords (subdominant to dominant to tonic). This sort of function theory does significant work merely by categorizing chords. For by grouping them into psychologically robust categories it makes important claims about how we hear the full range of possible diatonic progressions. Indeed, a function theory of this sort could be informative even if there were *no* functional regularities among tonal chord progressions: for by postulating psychologically real tonal functions, it asserts that we can categorize all possible diatonic chord progressions into a smaller set of perceptually similar groups.

The second way to think about functions does *not* postulate that they have psychological reality. On this view functions are mere contrivances, useful in that they simplify the rules that describe the permissible chord progressions. (This is the view taken in Dahlhaus 1968.) Consider for example, the following syntactic tonal chord sequences:

I-ii-V-I
I-IV-V-I
I-ii-vii°-I
I-IV-vii°-I

We can describe these four permissible tonal progressions using the single rule that chords can progress from tonic to subdominant to dominant to tonic. No assertion need be made about the psychological reality of chord functions; indeed, it may be that we hear these four progressions in completely different ways. Notice that in this sort of function theory, the grouping of chords into functional categories *cannot* be separated from the description of normative patterns of chord progression. For what justifies grouping V and vii° together as "dominant" chords, is simply the fact that both chords tend to move in similar ways.

Let us consider a function theory of the second type. We will ask to what extent we can group chords into functional categories on the basis of shared patterns of root motion. Returning to Example 9, we notice that the rows of the table can be used to define a "probability vector" that gives the chance that, in Bach's harmonic language, a chord of a given type will move to any other chord. Using the percentages from the first row of Example 9, we can see that the probability vector for the I chord is [23% 11% 0% 23% 31% 8% 6%]. We can consider *functions* to be resemblances between these vectors. Two chords that have the same function will tend to move to the same chords, with similar probabilities.

We can measure the similarities among these probability vectors using the common statistical measure known as the Pearson correlation coefficient.[21] Example 12 presents the correlations among these vectors. The two highest values indicate correlations among chords commonly thought to be functionally equivalent. There is an extremely strong correlation (of .98) between the vectors for V and vii°. This suggests that we have a reason for grouping these chords together as "dominant chords," solely on the basis of their tendencies to move similarly.

The next highest correlation is between ii and IV, both commonly considered "subdominant" triads. The correlation here, .774, is significantly lower than that between

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[21] The Pearson correlation coefficient, commonly called "correlation," measures whether there is a linear relationship between two variables. The value of a correlation ranges between –1 and 1. A correlation of 1 between two sets of values X and Y, means that there is an equation

$$Y = aX + b \qquad \text{(a and b constant, } a > 0\text{)}$$

that can be used to exactly predict each value of Y from the corresponding value of X. Thus, Y increases proportionally with X. Lower positive correlations indicate that the prediction of Y involves a greater degree of error. A negative correlation indicates that there is an equation

$$Y = aX + b \qquad (a < 0)$$

linking the variables. Thus, Y *decreases* as X gets larger. A correlation of 0 indicates that there is no linear relation between the quantities. When X is large, Y is sometimes large, and sometimes small.

V and vii° chords, and suggests that ii and IV behave quite differently. A glance at Example 9 shows why this is so. The IV chord has a much higher tendency to return to the I chord than does the ii chord. (The figures are 24% for the IV-I progression as compared to 8% for ii-I.) This is, in fact, the major reason why ii and IV are less closely correlated than vii° and V: if we were to reduce the IV-chord's tendency to move to I to 8% (equivalent to the ii chord's tendency to move to I) then the correlation between IV and ii would leap to the very high .96.

Interestingly, there is a tradition in music theory that helps us interpret this fact. Following Nadia Boulanger, Robert Levin has articulated the view that the IV chord possesses two distinct tonal functions: a "plagal" function associated with the IV chord's tendency to move to I, and a "predominant function" associated with its tendency to move otherwise. We can express this idea in our more quantitative terms by saying that the probability vector for the IV chord can be decomposed into two independent vectors representing two different tonal functions:

	IV		[24 12 2 10 29 4 18]
=		=	
	"plagal IV"		[16 0 0 0 0 0 0]
+			
	"predominant IV"		[8 12 2 10 29 4 8]

Again, it is suggestive that there is a very high correlation (.96) between the "predominant" component of IV's behavior and the probability vector for the ii chord. This suggests grouping the IV and ii together—with the proviso that the IV chord also participates in distinctive, "plagal" motions.

Let us now try to use this method to verify the assertions that the iii chord can function as both tonic and dominant, and that the vi chord can function as both predominant and tonic. The natural way to interpret these proposals is to try to correlate the probability vectors associated with iii and vi chords with linear combinations of vectors representing their proposed functions. Thus it would be interesting if there were some positive numbers a and b such that

$$\mathrm{iii}_v =_c a\mathrm{I}_v + b\mathrm{V}_v \qquad \text{or} \qquad \mathrm{vi}_v =_c a\mathrm{I}_v + b\mathrm{ii}_v$$

(Here the subscript "v" indicates the vector associated with the relevant chord; and the symbol "$=_c$ " should be read as "is very highly correlated with.") The above equations express the thought that the vector associated with the iii chord is extremely highly correlated with some mixture of the vectors associated with I and V, and the vector associated with the vi chord is highly correlated with some mixture of the vectors associated with ii and I.

Unfortunately, there are no positive numbers a and b that produce a correlation of the sort desired. Some care must be taken in interpreting this fact. Correlation, useful though it is, measures only one type of relationship, and it is particularly unsuited to capturing our intuitions about the relationships among relatively even probability distributions.[22] For this reason, we should be careful not to think we have "refuted" Riemann's theory of the iii and vi chord. At the same time, our failure may lead us to wonder about the viability of Riemann's functional classifications. Is it helpful to think of the iii chord as being both "tonic" and "dominant"? Is the vi chord both "subdominant" and "tonic"? Or should we instead understand these chords as independent entities, functionally *sui generis?*

There are two issues here. The first is that Riemann classifies as functionally similar chords which, in Bach's chorales, typically participate in very different sorts of chord motion. For Riemann, IV and ii are both subdominant chords, but I-IV-I is common while I-ii-I is not. Likewise, Riemann classifies iii and V as dominant chords, but IV-V-I is common while IV-iii-I is not. Thus we cannot identify the syntactical chord progressions in functional terms alone. Instead, we need to add additional, chord-specific

[22] The correlation between the vector [1 0 0] and [.34 .33 .33] is 1, even though the former represents a maximally uneven distribution of probabilities, while the latter is very even. Conversely, the correlation between [.34 .33 .33] and [.33 .33 .34] is -.5, even though these two distributions are both very even. For this reason, I consider arguments based on statistical correlation to be at best suggestive.

principles which distinguish between functionally identical progressions. The second difficulty is that the vi and iii chords possess multiple functions, so that it is not always clear how to evaluate chord progressions in functional terms. Is the progression iii-IV a typical T-S progression or an nonstandard D-S progression? Does a rule permitting T-S-T progressions justify the use of I-vi-I? Function theorists are not always explicit about how to decide such questions. This again means that Riemann's categories are not sufficient for identifying the commonly-used chord progressions.

In light of this, it seems reasonable to conclude that one cannot defend Riemann's functional categories without attributing psychological reality to functions.[23] For if one treats functions as mere conveniences, useful for simplifying the description of the syntactical progressions of tonal harmony, then one is forced to conclude that Riemann's categories are overly broad. There are, to be sure, good reasons for grouping V and vii° as "dominant" chords (since both overwhelmingly tend to move to the tonic), and for grouping ii and IV as "predominants" (since both tend to progress to dominant harmonies). But there are not the same strong reasons for classifying the iii and vi chords as part of larger functional groups. Different theorists have responded to this difficulty in different ways. Some, like Kostka and Payne, adopt a hybrid view, using smaller functional categories to associate ii and IV, and V and vi, while treating iii and vi as independent entities—much as a scale-degree theorist would. Others, such as Agmon, have retained Riemann's categories, attempting to justify them in cognitive and psychological terms.

I will not consider this second approach, as my concern here is simply with the attempt to provide an efficient grammar of elementary major-mode harmony. It is worth noting, however, that the first approach represents an extremely small modification to the scale-degree theory considered above. For if one interprets functions simply as similarities among chord-tendencies, rather than in substantive psychological or metaphysical terms, then there is hardly any difference between scale-degree and function theorists. A "pure" scale-degree theorist would assert that the triads on each of the seven scale degrees are independent entities, each behaving in its own characteristic

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[23] Dahlhaus 1968 tries to defend both of these theses simultaneously.

way. The function view we have been considering merely adds that some of these chords behave in similar enough ways to justify grouping them together in categories. It is hard to imagine why a scale-degree theorist would want to deny this.

3. Conclusion.

Of the three views we have considered, the scale-degree theory, implemented as a first-order Markov model, yields the best grammar of elementary tonal harmony. The root-motion theory is too restrictive: while it captures an important subset of the tonal progressions (the T-S-D-T progressions), it cannot adequately explain the prevalence of I-V-I and I-IV-I progressions. More generally, its commitment to scale-degree symmetry means it cannot account for the highly asymmetrical "subdominant" progressions. By contrast, an expansive function theory—one which upholds Riemann's functional categories, and which attempts to identify the syntactic chord progressions in functional terms alone—has proved to be overly permissive. For this kind of theory does not have the resources to explain the differences between functionally identical progressions such as I-IV-I and I-ii-I. The scale-degree model exemplified by Example 10 strikes a good middle ground, capturing a large number of syntactic progressions without producing many erroneous progressions. Furthermore, the scale-degree model incorporates many of the important insights from the other two theories. As we have seen, it has many of the features that root-motion theorists take to define tonal harmony: it exhibits root-motion asymmetry, generating more "dominant" than "subdominant" progressions, and permits the full range of dominant progressions on many scale-degrees. The scale-degree model also suggests a restricted sort of functionalism, one which groups ii and IV together as "subdominant" chords, and V and vii° as "dominants."

There are, of course, problems with the model. The fact that it has no memory means that it is liable to produce repetitive sequences and to make inappropriate use of plagal progressions. It cannot account for some of the subtler features of elementary tonal syntax, such as inversion-specific and other "idiomatic" progressions. But these problems are all relatively tractable. It would be fairly easy for someone, interested in exploring artificial intelligence models of elementary diatonic harmony, to write a computer program that corrected these difficulties. Such a program would essentially encode the higher-level principles internalized by human musicians—principles like "avoid unmotivated repetition," and "the cadential six-four is most common at the end of a phrase."

It is instructive to consider one important way in which the Markov model does *not* fail. Noam Chomsky (1958) famously demonstrated that natural languages cannot be modeled by finite-state Markov chains. The basic idea is that natural languages permit a kind of recursive, hierarchical structuring that demands a similarly recursive grammar. For example, the simple sentence

> 1) The man bought a dog.

can be used to form an infinite variety of longer sentences of potentially limitless complexity. One can embellish it with dependent phrases that can themselves contain whole sentences:

> 2) **The** blind, one-legged **man** who owned the car that ran over my little brother's favorite bicycle **bought a** mangy, unkempt, flea-bitten **dog,** which barked like a hyena.

We can also embed it as a component of longer sentences:

> 3) Either **the man bought a dog** or his wife bought it.

> 4) Greg, Peter, and **the** other **man bought** a bicycle, a boat, and **a dog**, respectively.

An adequate grammar of English needs to express the fact that phrases and sentences form syntactic units that can be recursively combined. To do so, it must have capabilities that go beyond those of a simple finite-state probabilistic Markov model. (In Chomsky's parlance, it must be a "Type 2" rather than a "Type 3" grammar.)

Schenkerian theorists sometimes suggest that musical grammar has a similar sort of recursive complexity.[24] The idea is that a simple chord progression such as

5) I-V-I

Can be embellished with numerous subsidiary (or "prolongational" progressions):

6) **I-**V^6-I-I^6-ii^6-**V-I**

Orthodox Schenkerians see these hierarchical embeddings as extending across very large spans of time. Indeed, it is typical to analyze whole movements as "prolonging" (or embellishing) a single fundamental (or "background") I-V-I chord progression.[25]

Notice, however, that there is a crucial difference between the hierarchical structures in natural language and those we purportedly find in elementary tonal harmony. The harmonic progression (6) can be analyzed as a concatenation of two perfectly syntactical progressions:

I-V^6-I and I^6-ii^6-V-I

By contrast, sentences (2)—(4) cannot be analyzed as a concatenation of grammatically well-formed subsentences. Thus in the natural language case, we are *required* to postulate a hierarchical grammar in order to account for our most basic intuitions about grammaticality. This is not true in the musical case. Tonal harmony generally consists in

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[24] For example, Salzer (1982, 10-14) raises a complaint about Roman-numeral analysis that is in some ways parallel to Chomsky's criticism of finite-state Markov chains.

[25] Note that there is a vast difference in scale between the hierarchies of Chomskian linguists and those of Schenkerian analysts. For linguists, hierarchical structuring typically appears in single sentences. For Schenkerians, hierarchical structuring applies to the length of entire musical movements, which tend to be several orders of magnitude longer than single sentences. This reflects the fact that Schenkerian theory was born out of nineteenth-century ideas about the "organic unity" of great artworks: in demonstrating that great tonal works prolong a single I-V-I progression, Schenker took himself to be demonstrating that these works were organic wholes.

a concatenation of relatively short, well-formed chord progressions, each of which tends to express clear T-(S)-D-T functionality.[26]

Where this leaves us is an open question. Those who favor a concatenationist approach may feel that this demonstrates that music does not possess anything as complex as a "grammar." If we can, indeed, model tonal harmonies with something like a finite-state Markov model, then this just shows how far music is from the rich structures of natural language. Others may feel that music does display complex hierarchical structure akin to that of natural language, but that this structure is not manifested by the harmonic progressions alone. Instead, hierarchy in music will be conveyed—as Schenker asserted it was—by details of rhythm, phrasing, and register. (Some Schenkerians have even argued that the very attempt to consider harmony in isolation from counterpoint, as I have done in this paper, involves a profound methodological mistake.[27]) I will not attempt to settle this matter. But I will say that recent critics have overstated the case against the scale-degree perspective. For as we have seen, the theory provides a fairly good model for elementary diatonic harmony—a nearly adequate grammar, whose basic principles are amply confirmed by empirical evidence. While scale-degree theories may not represent the last word in harmonic thinking, they surely form an important component of any adequate theory of tonal harmony.

[26] Typically, these individual progressions will vary in their perceived strength or importance: some (like the ii^6-V-I progression in [6]) may be felt to be more conclusive than others. But this does not in itself compel us to adopt a hierarchical picture. After all, the sentences in a well-written paragraph of English differ in their weight and perceived importance. But linguists do not tend to assert hierarchical structures that extend across sentence boundaries.

[27] See Beach 1974 for polemical comments to this effect. My own view is that the data presented in this paper shows that tonal harmonies have a clear structure, even when considered in isolation. One wonders: would Beach assert that it is mere coincidence that tonal music tends to involve a small number of recurring harmonic patterns?

APPENDIX: The 30 Chorales used in this study.

Title	**Key**	**Riemenschnieder**	**BWV**	**Breitkopf/ Kalmus**
O Welt, ich muß dich lassen	A-flat	117	244.10	294
Meinen Jesum laß ich nicht	E-flat	299	380	242
O Mensch, bewein dein' Sünde groß	E-flat	306	402	286
Herr Christ, der einge Gottessohn	B-flat	101	164.6	127
Ich dank dir, lieber Herre	B-flat	272	348	177
Jesu, meiner Freuden Freude	B-flat	350	360	364
Wenn wir in höchsten Nöten sein	F	68	431	358
Erstanden ist der heilige Christ	F	176	306	85
Herr Christ, der einge Gottessohn	F	303	96.6	128
Wie schön leuchtet der Morgenstern	F	323	172.6	376
Hilf, Herr Jesu, laß gelingen 2	F	368	248(4).42	
Christus ist erstanden	C	200	284	51
Ich dank dir Gott für alle Wohltat	C	223	346	175
Nun lob, mein Seel, den Herren	C	268	389	269
Wie nach einer Wasserquelle	C	282	25.6	
Aus meines Herzens Grunde	G	1	269	30
Wie nach einer Wasserquelle	G	67	39.7	104
Komm, heiliger Geist, Herre Gott	G	69	226.2	221
Der Tag, der ist so freudenreich	G	158	294	62
Es ist das Heil uns kommen her	G	248	117.4	90
Liebster Jesu, wir sind hier	G	328	373	228
Ermuntre dich, mein schwacher Geist	G	361	248(2).12	80
Valet will ich dir geben	D	24	415	314
Herzlich tut mich verlangen	D	98	244.15	163
Die Wollust dieser Welt	D	255	64.4	280
Ich dank dir, lieber Herre	A	2	347	176
Nun danket alle Gott	A	32	386	257
Ach bleib bei uns, Herr Jesu Christ	A	177	253	1
O Welt, sieh hier dein Leben	A	366	394	290
Es ist das Heil uns kommen her	E	290	9.7	87

BIBLIOGRAPHY

Aldwell, Edward and Schachter, Carl. 2002. *Harmony and Voice Leading*. 3rd edition. Belmont: Wadsworth.

Agmon, Eytan. 1995. "Functional Harmony Revisited: A Prototype-Theoretic Approach." *Music Theory Spectrum* 17:2, 196-214.

Beach, David. 1974. "The Origins of Harmonic Analysis." *Journal of Music Theory* 18.2, 274-30

Chomsky, Noam. 1958. *Syntactic Structures*. The Hague: Mouton.

Dahlhaus, Carl. 1968. *Untersuchungen über die Entstehung der harmonischen Tonalität*. Kassel: Bärenreiter.

Kostka, Stefan and Payne, Dorothy. 2000. *Tonal Harmony*. Fourth Edition. New York: Alfred A. Knopf.

Meeus, Nicolas. 2000. "Toward a Post-Schoenbergian Grammar of Tonal and Pre-tonal Harmonic Progressions." *Music Theory Online* 6:1.

Rameau, Jean Paul. 1722. *Traité de l'harmonie,* Paris: Ballard. Translated by Philip Gossett as *Treatise on Harmony*. New York: Dover, 1971.

Riemann, Hugo. 1893. *Vereinfachte Harmonielehre*. London: Augener.

Schoenberg, Arnold. [1954] 1969. *Structural Functions of Harmony* . Edited by Leonard Stein. New York: Norton.

Sadai, Yizhak. 1980. *Harmony in its Systemic and Phenomenological Aspects*. Jerusalem: Yanetz.

Salzer, Felix. [1961] 1982. *Structural Hearing*. New York: Dover.

Vogler, Georg. 1776. *Tonwissenschaft und Tonsezkunst*. Mannheim, Kurfürstliche Hofbuchdruckerei.

Weber, Gottfried. 1817-21.*Versuch einer gordneten Theorie der Tonsetzkunst*. 3 vols. Mainz: B. Schott.

Example 1. Meeus's classification of tonal chord progressions

CATEGORY	MAIN PROGRESSION	SUBSTITUTES
Dominant	A fifth down	A third down or a second up
Subdominant	A fifth up	A third up or a second down

Example 2. Diatonic triads in descending-third sequence

I -> vi -> IV -> ii -> vii° -> V -> **[iii]** -> I

Example 3. Progressions produced by the root-motion model

PROGRESSION	FUNCTIONAL TYPE
I-ii-V-I	T-S-D-T
I-ii-vii°-I	T-S-D-T
I-ii-vii°-V-I	T-S-D-T
I-IV-V-I	T-S-D-T
I-IV-vii°-I	T-S-D-T
I-IV-vii°-V-I	T-S-D-T
I-IV-ii-V-I	T-S-D-T
I-IV-ii-vii°-I	T-S-D-T
I-IV-ii-vii°-V-I	T-S-D-T
I-vi-vii°-I	T-PS-S-D-T
I-vi-vii°-V-I	T-PS-S-D-T
I-vi-ii-V-I	T-PS-S-D-T
I-vi-ii-vii°-I	T-PS-S-D-T
I-vi-ii-vii°-V-I	T-PS-S-D-T
I-vi-IV-V-I	T-PS-S-D-T
I-vi-IV-vii°-I	T-PS-S-D-T
I-vi-IV-vii°-V-I	T-PS-S-D-T
I-vi-IV-ii-V-I	T-PS-S-D-T
I-vi-IV-ii-vii°-I	T-PS-S-D-T
I-vi-IV-ii-vii°-V-I	T-PS-S-D-T

T = tonic, PS = pre-subdominant; S = subdominant, and D = dominant

Example 4. Root progressions in Bach and Palestrina

a) in Bach chorales

	DOWN	UP
FIFTH	**1842 (35%)**	510 (10%)
THIRD	**682 (13%)**	533 (10%)
SECOND	318 (6%)	**1354 (26%)**

5240 total progressions, of which 74% are "dominant."

b) in Palestrina

	DOWN	UP
FIFTH	**319 (28%)**	168 (14%)
THIRD	**253 (22%)**	152 (13%)
SECOND	91 (8%)	**176 (15%)**

1159 total progressions, of which 65% are "dominant."

Example 5. Chord progressions in Bach chorales, categorized according to functional type

1.T-D-T			63 progressions
I-V-I	S-D	59	
I-vii°-I	S-D	4	
2. T-S-D-T			53 progressions
I-IV-V-I	D-D-D	15	
I-ii-V-I	D-D-D	15	
I-ii-I_{c}°-V-I	D-D-D	1	
I-ii-vii°-I	D-D-D	11	
I-IV-vii°-I	D-D-D	7	
I-IV-ii-V-I	D-D-D-D	2	
I-IV-ii-vii°-I	D-D-D-D	1	
I-IV-ii- I_{c}°-V-I	D-D-D-D	1	
3. T-S-T			18 progressions
I-IV-I	D-S	18	
4. Progressions involving vi or IV^{6}			
a. vi as pre-predominant, as bass arpeggiation, and as predominant			11 progressions
I-vi-ii-V-I	D-D-D-D	6	
I-vi-IV-V-I	D-D-D-D	1	
I-vi-IV-ii-V-I	D-D-D-D-D	1	
I-vi-I^{6}-V-I	D-S-S-D	1	
I-vi-V-I	D-S-D	2	
b. vi and IV^{6} as part of a deceptive progression			8 progressions
I-V-vi-IV-vii°-I	S-D-D-D-D	3	
I-V-vi-IV-V-I	S-D-D-D-D	1	
I-V-vi-I^{6}-V-I	S-D-S-S-D	1	
I-IV-V-vi-I^{6}-V-I	D-D-D-D-S-S-D	1	
I-IV-V-vi-I_{c}°-V-I	D-D-D-S-D	1	
I-vi-IV-V-IV^{6}-I_{c}°-ii^{6}-V-I	D-D-D-S-S-D-D-D	1	
c. vi and IV^{6} expanding V			9 progressions
I-V-IV^{6}-vii°7-I	S-S-D-D	3	
I-V-IV^{6}-V^{6}-I	S-S-D-D	1	
I-IV-V-IV^{6}-vii°7-I	D-D-S-D-D	1	
I-IV-V-vi-vii°[5/3]-I	D-D-D-D-D	1	
I-IV-V-IV^{6}-V^{6}-I	D-D-S-D-D	1	
I-IV-V-IV^{6}-I-V	D-D-S-S-D	1	
I-V^{6}-vi^{6}-vii°^{6}-I^{6}	S-D-D-D	1	

5. V^6 initiating stepwise descent in the bass 3 progressions

I-V^6-IV^6-vi-ii^6-V-I	S-S-S-D-D-D	1
I-V^6-vi-V^6-I	S-D-S-D	1
I-V^6-vi-I^6-ii^6-V-IV^6-vii°-I	S-D-S-D-D-S-D-D	1

6. Progressions involving iii 2 progressions

I-IV^6-iii-vi-ii-vii°-I	D-S-D-D-D-D	1
I-vi-iii-IV-I-ii^6-V-I	D-S-D-S-D-D-D	1

7. Strange progressions 2 progressions

I-IV-iii-IV-V-I	D-S-D-D-D	1 *(derives from I-IV-*$I^{O}_{¿}$*-V-I)*
I-IV-vii°-IV^6-I	D-D-S-S	1 *(IV*^{6}*-I harmonizes a suspension)*

Example 6. Dominant chord progressions which do not appear in Example 5

a) Progressions involving iii

ii–iii
V–iii
vii°–iii
iii–I

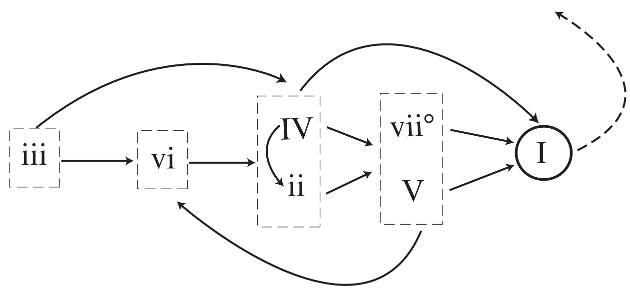
b) Other progressions

vii°–V

Example 7. Subdominant progressions appearing in Example 5

Progression Type	**Number of Appearances**
I-V	75 (63%)
IV-I	19 (16%)
V-IV^6	10 (8%)
I-vii°	4 (3%)
vi-I^6	4 (3%)
vi-V	3 (3%)
vi-iii	1 (1%)
IV^6–vi	1 (1%)
IV^6-iii	1 (1%)
vii°-IV^6	1 (1%)

Example 8. Kostka and Payne's map of major-mode harmony



Example 9. Scale-degree progressions in the Bach chorales

	I	**ii**	**iii**	**IV**	**V**	**vi**	**vii°**
I	73 (23%)	36 (11%)	1 (0%)	74 (23%)	99 (31%)	26 (8%)	6 (2%)
ii	7 (8%)	12 (14%)	1 (1%)	2 (2%)	39 (45%)	5 (6%)	20 (23%)
iii	0 (0%)	4 (20%)	1 (5%)	5 (25%)	1 (5%)	8 (40%)	1 (5%)
IV	33 (24%)	16 (12%)	3 (2%)	14 (10%)	40 (29%)	5 (4%)	25 (18%)
V	174 (67%)	2 (1%)	3 (1%)	11 (4%)	40 (15%)	29 (11%)	0 (0%)
vi	10 (11%)	19 (22%)	5 (6%)	16 (18%)	18 (21%)	9 (10%)	10 (11%)
vii°	43 (81%)	0 (0%)	2 (4%)	3 (6%)	3 (6%)	2 (4%)	0 (0%)

Example 10. A simple Markov model of tonal harmony

a) the matrix used by the model

	I	ii	iii	IV	V	vi	vii°
I	0%	14%	1%	30%	41%	11%	3%
ii	0%	0%	0%	0%	61%	8%	31%
iii	0%	0%	0%	86%	0%	14%	0%
IV	29%	14%	0%	0%	35%	0%	22%
V	86%	0%	0%	0%	0%	14%	0%
vi	0%	31%	0%	25%	29%	0%	15%
vii°	100%	0%	0%	0%	0%	0%	0%

b) progressions produced by the model

1.T-D-T			71 progressions
I-V-I	S-D	69	
I-vii°-I	S-D	2	
2. T-S-D-T			59 progressions
I-IV-V-I	D-D-D	20	
I-ii-V-I	D-D-D	16	
I-ii-vii°-I	D-D-D	6	
I-IV-vii°-I	D-D-D	11	
I-IV-ii-V-I	D-D-D-D	5	
I-IV-ii-vii°-I	D-D-D-D	1	
3. T-S-T			13 progressions
I-IV-I	D-S	13	
4. Progressions involving vi			
a. vi as pre-subdominant and as subdominant			9 progressions
I-vi-ii-V-I	D-D-D-D	2	
I-vi-IV-V-I	D-D-D-D	1	
I-vi-vii°-I	D-D-D	2	
I-vi-V-I	D-S-D	4	
b. vi as part of a deceptive progression			12 progressions
I-V-vi-vii°-I	S-D-D-D	2	
I-V-vi-ii-V-I	S-D-D-D-D	2	
I-V-vi-IV-V-I	S-D-D-D-D	1	
I-V-vi-IV-vii°-I	S-D-D-D-D	1	
I-V-vi-IV-ii-vii°-I	S-D-D-D-D-D	1	
I-V-vi-V-I	S-D-S-D	1	

I-ii-V-vi-ii-V-I	D-D-D-D-D-D	1
I-IV-V-vi-IV-V-I	D-D-D-D-D-D	1
I-IV-V-vi-V-I	D-D-D-S-D	1
I-V-vi-ii-vi-IV-V-I	D-D-D-S-D-D-D	1

5. Problematic progressions 5 progressions

a. repetitive progressions

I-ii-V-vi-IV-V-vi-V-I	D-D-D-D-D-D-S-D	1
I-vi-V-vi-V-I	D-S-D-S-D	1

b. IV-I occurring late in the progression

I-vi-IV-I	D-D-S	2
I-V-vi-IV-I	S-D-D-S	1

Example 11. Asymmetry between dominant and subdominant progressions, expressed as a function of chord-type

a)

	Dominant Progressions	**Subdominant Progressions**
V	94%	6%
vii°	91%	9%
ii	81%	19%
iii	68%	32%
IV	66%	34%
vi	58%	42%
I	56%	44%

b)

I (56%) -> vi (58%) ->IV (66%) -> ii (81%)-> vii° (91%) -> V(94%)
[iii 68%]

Example 12. Correlations among diatonic probability-vectors.

	I	**ii**	**iii**	**IV**	**V**	**vi**	**vii°**
I	1.000	0.440	-0.110	0.686	0.475	0.646	0.376
ii	0.440	1.000	-0.373	0.774	-0.042	0.502	-0.170
iii	-0.110	-0.373	1.000	-0.604	-0.370	0.135	-0.422
IV	0.686	**0.774**	-0.604	1.000	0.511	0.451	0.434
V	0.475	-0.042	-0.370	0.511	1.000	-0.137	0.980
vi	0.646	0.502	0.135	0.451	-0.137	1.000	-0.200
vii°	0.376	-0.170	-0.422	0.434	**0.980**	-0.200	1.000