CHAPTER 8

DUALISM AND THE BEHOLDER’S EYE: INVERSIONAL SYMMETRY IN CHROMATIC TONAL MUSIC

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“The importance of symmetry in modern physics,” writes Anthony Zee, “cannot be overstated.” Zee alludes to the fact that some of the most celebrated discoveries in the history of science—from Galileo’s law of inertia to Einstein’s principle of relativity and Dirac’s prediction of antimatter—involve the realization that physical laws possess unexpected symmetries. These symmetries allow us to change our description of the world (for instance, by using different numbers to refer to locations in space) without altering the form of our laws. The laws, by virtue of being insensitive to these changes, are symmetrical with respect to them.

Though it is not immediately obvious, this notion of symmetry plays a similarly central role in music theory as well. Indeed some of the most important developments in the history of theory—including Rameau’s root functionality, Weber’s Roman numerals, and Oettingen’s and Riemann’s dualism—involve claims that the musical universe possesses unexpected symmetries. Music theorists, however, are less explicit about the topic than physicists: they typically propose symmetries en passant, by developing notation and terminology that is invariant under the relevant musical transformations. For example, when theorists say that a note is “an F♯,” the description remains true even if the note is transposed by one or more octaves. The term “F♯” thus embodies a symmetry (octave equivalence) by virtue of being insensitive to a musical operation (octave transposition)—much as the laws of
Newtonian physics remain the same whether one chooses to describe oneself as being at rest or in motion with a constant velocity.\(^2\)

In what follows, I will consider the theme of inversional symmetry as it manifests itself in Riemann’s theoretical writings and in late-nineteenth-century chromatic music. Section 1 provides historical background. I begin with Rameau, who proposed that the laws of tonal harmony are invariant under four basic operations: reordering, octave shift, note duplication, and chromatic transposition. Weber’s Roman numeral notation, which develops and fulfills Rameau’s ideas, is symmetric under two additional operations: diatonic transposition and what I call triadic extension. I argue that traditional tonal syntax does indeed manifest these symmetries, at least to a good first approximation. Section 2 describes Riemann’s “dualism” as an attempt to incorporate inversion into the Rameau/Weber collection of symmetries. As many commentators have noted, dualism is unsatisfactory because traditional tonal syntax is not in fact inversionally symmetric. Section 3 then asks whether the “second practice” of nineteenth-century chromaticism involves inversional symmetry.\(^3\) I argue that it does, but only because inversional relationships arise as necessary by-products of a concern with efficient voice leading. Section 4 contrasts this view with a more orthodox, Riemannian understanding of dualism. Finally, section 5 illustrates my contrapuntal approach by analyzing a Brahms intermezzo.

1. RAMEAU AND WEBER

Let us begin by defining a “basic musical object”—the atom of music-theoretical discourse—as an ordered sequence of pitches.\(^4\) Basic musical objects can be ordered in time or by instrument: \((C_4, E_4, G_4)\) can represent an ascending C major arpeggio played by a single instrument or a simultaneous chord in which the first instrument plays \(C_4\), the second instrument plays \(E_4\), and the third \(G_4\).\(^5\) (Instruments can be labeled arbitrarily: what matters is simply that they are distinguishable somehow.) A progression is an ordered sequence of musical objects: thus, \((C_4, E_4, G_4) \rightarrow (C_4, F_4, A_4)\) is a progression with \((C_4, E_4, G_4)\) as its first object and \((C_4, F_4, A_4)\) as its second.

Basic musical objects are uninteresting because they are so particular: until we decide how to group objects into categories, \((C_4, E_4, G_4)\) remains unrelated to \((E_4, C_4, G_4)\).\(^6\) Theorists typically classify musical objects by defining musical transformations that leave objects “essentially unchanged.” For example, \((C_4, E_4, G_4)\) can be transformed in three ways without modifying its status as a C major chord: its notes can be reordered, placed into any octave, or duplicated. This process of reordering, octave shift, and note duplication can be reiterated to produce an endless collection of objects, all equally deserving of the name “C major”: \((C_4, E_4, G_4), (E_4, C_4, G_4, G_4), (G_4, G_4, G_4, E_2, C_6)\), and so on. “C majorishness” is therefore determined by an
object’s pitch-class content, rather than the order or register in which its pitches are stated. We can say the concept “chord” embodies the symmetries of octave equivalence, reordering, and note duplication.

Rameau is often considered the first theorist to articulate the modern conception of a chord, determining the harmonic identity of groups of notes by their pitch-class content alone. He also classified chords into larger categories, using terms like “major [perfect] chord” and “minor [perfect] chord” to refer to what we would call transpositional set classes. These more general terms are invariant under a larger group of symmetry operations: we can reorder, shift octaves, duplicate notes, or transpose every note by the same amount, all without changing an object’s status as a major chord. “Major chordishness” is thus determined not by the specific pitch classes in an object, but by the intervals between them. These intervals, and hence “major chordishness,” are preserved under transposition.

Underlying Rameau’s classificatory innovations was a third and more far-reaching suggestion: that chords provide the appropriate vocabulary for formulating the basic laws of tonal harmony. Thus, if example 8.1a is an acceptable harmonic progression, then so is example 8.1b, since the two passages contain exactly the same series of chords. Furthermore, the laws of harmony are on Rameau’s view transpositionally invariant: thus if example 8.1a is acceptable in C major, then example 8.1c should be acceptable in G major. (The principles of functional harmony, in other words, do not change from key to key.) We can say that the fundamental harmonic laws are invariant under octave shifts, reordering, note duplication, and transposition.

At this point, I should pause to explain that there are actually two different ways in which a symmetry can operate upon a sequence of chords: individual symmetries can be applied independently to the objects in a progression, while uniform symmetries must be applied in the same way to each object. Example 8.1 illustrates. Reordering, octave shift, and note duplication are individual symmetries, and can be applied differently to each chord without changing the progression’s fundamental harmonic character. (This process may create awkward voice leading, but that is a separate matter.) By contrast, transposition is a uniform symmetry: one must use a single transposition when shifting music into a new key. Example 8.1d illustrates.

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Ex. 8.1. Symmetry in traditional harmonic analysis.

(a) (b) (c) (d)

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the disastrous result of applying transposition individually, with the nonfunctional sequence $E_b$ major $\rightarrow$ F major $\rightarrow$ B major bearing little resemblance to the original. $^{11}$ Clearly, theorists of tonal music want to emphasize the relation between examples 8.1a–c, all of which are I–IV–V–I progressions; the relationship between these and example 8.1d, though significant in some twentieth-century contexts, is not important in traditional tonality.

Although Rameau invented many features of modern harmonic theory, he did not devise a fully satisfactory notation for progressions. This was accomplished by Gottfried Weber almost a hundred years later. Weber’s Roman numerals are invariant under (uniform) transposition and (individual) reordering, octave shift, and note duplication—precisely as Rameau’s theory requires. (Note in particular that the same Roman numerals apply to examples 8.1a–c, but not example 8.1d.) But Weber’s notation also encodes two additional symmetries not intrinsic to Rameau’s theory. The first might be called the triadic extension symmetry: as shown in examples 8.2a–b, it is possible to extend a diatonic “stack of thirds” upward without changing its Roman numeral. This symmetry permits us to group together collections with different pitch class content—for example, to treat $\{G, B, D\}$, $\{G, B, D, F\}$, and $\{G, B, D, F, A\}$ as versions of the V chord in C major. $^{12}$ (Triadic extension is an individual symmetry, since sevenths can be added to only some of the chords in a progression.) The second symmetry is diatonic transpositional symmetry: as shown in examples 8.2a and 8.2c, it is possible to transpose a passage of music diatonically, between relative major and minor, without changing its Roman numerals. $^{13}$ (Since traditional tonality uses only two modes, the action of diatonic transposition is restricted to shifts between relative major and minor.) Like chromatic transposition, diatonic transposition is a uniform symmetry: we would radically transform the sense of a passage if we were to diatonically transpose only some of its chords.

At first blush, diatonic transpositional symmetry may seem pedestrian. But it is interesting to note that there are broadly tonal styles displaying no such symmetry. For example, in rock, different modes draw on different repertoires of chord progressions: i–VII–vi–VII–i and i–III–vi–VII–i are common in minor, while the analogous major-mode progressions (I–viiº–vi–viiº–I and I–iii–vi–viiº–I) are extremely rare. In some sense, this is to be expected: diatonic transposition, by changing the quality of the triads on each scale degree, changes the sound of the various diatonic chord progressions, and one would not necessarily expect musical syntax to be

Ex. 8.2. Two additional symmetries. The second progression relates to the first by triadic extension, while the third relates to the first by diatonic transposition.
insensitive to these changes. To my mind, it is somewhat remarkable that classical harmony exhibits such a high degree of symmetry between major and minor. This symmetry permits the extraordinary expressive effect of presenting “the same” musical material in very different affective contexts.

The Rameau/Weber theory, of course, is merely an approximation to actual tonal practice; a more accurate theory can be obtained by combining Roman numerals with figured-bass notation. This hybrid system, which is nearly universally accepted by American pedagogues, allows theorists to make very refined observations about harmonic motion—for instance, that the chord progression V–IV is common while the progression V–IV is rare. The hybrid system must be further extended with purely contrapuntal principles, such as the emphasis on efficient voice leading and the prohibition of parallel perfect fifths. In this sense, the Rameau/Weber theory describes only some of the conventions of traditional tonal music. In my view this is no flaw: tonality is extraordinarily complex, and we should not ask any one theory to describe it completely. Rather than focus on the inadequacies of traditional theory, I prefer to marvel at the fact that it provides an extraordinarily efficient description of harmonic patterns found in a wide range of tonal music—even music written by composers trained in the earlier figured-bass tradition. Its simplicity and scope surely qualify it as one of the greatest achievements of Western music theory.

2. Dualism and Traditional Tonality

This brings us to the “dualism” of Oettingen and Riemann, which can be understood as an attempt to augment the Rameau/Weber symmetries with (uniform) inversional equivalence. Like Rameau and Weber, Riemann articulated his theory by developing analytical terms that are invariant under his favored musical transformations: thus one can often use precisely the same Riemannian description to describe inversionally related passages. The interesting question is whether traditional tonal practice requires this sort of inversionally symmetrical terminology.

Though Riemann’s dualistic thinking was guided by dubious forays into metaphysics and acoustics, it can be purified of such connections. What is more important, from a modern perspective, is that inversion and transposition are the only distance-preserving transformations of pitch and pitch-class space (example 8.3a). Since distance is a fundamental musical attribute, there are a number of theoretical contexts where it is useful to think dualistically: for example, when cataloguing tertian sonorities, or triadic progressions containing common tones, or the efficient voice-leading possibilities between set classes of a given type. We will return to this point momentarily.

Riemann, following Oettingen, conceived the minor-key system as the inversion, rather than the diatonic transposition, of major: thus, as shown in example 8.3b, he...
called C, E, and G the root, third, and fifth of the C major triad, while labeling G, Eb, and C the root, third, and fifth (respectively) of C minor. This “dualistic” terminology is invariant under inversion, which sends the root of a major triad to the Riemannian “root” of a minor triad, the third to the third, and so on (example 8.3b).

Riemann also developed an inversionally symmetrical vocabulary for classifying progressions: thus a single Riemannian term, Gegenquintschritt, describes both the progression C major → F major and its inversion, C minor → G minor (example 8.3c). Though Riemann devised names only for triadic progressions—the so-called Schritte and Wechsel—we can easily extend this idea: let us say that two progressions are dualistically equivalent if they are related by (uniform) transposition or inversion. For example, the progression A♭7 → C major, used in the standard resolution of the German augmented sixth, is dualistically equivalent to Fø7 → E minor, the penultimate progression in Tristan. This is because the inversion that transforms A♭7 to Fø7 also transforms C major to E minor. In much the same way, the chord progression from C augmented to C diminished seventh (or Caug → Cø7) is dualistically equivalent to Cø7 to Cø7, since one of the inversions that transforms Cø7 to Cø7 leaves Cø7 invariant. These definitions allow us to determine whether any two progressions are dualistically equivalent or not.

So is traditional tonal syntax invariant under inversion? Did Riemann, like Rameau, manage to describe a symmetry of traditional tonal chord progressions? Theorists generally agree that the answer is “no.” The harmonic patterns in traditional tonal music do not exhibit even approximate signs of inversional invariance: though I–IV–V–I is very common in tonal music, its inversion i–v–iv–i is extremely rare. In this respect, traditional theory is correct in describing major and minor as being related by diatonic transposition rather than by inversion. Example 8.4 demonstrates. We begin with a standard I–ii♭5–V–I progression in major. Example 8.4b transposes this pattern downward by two diatonic steps, raising the leading
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tone G → G♯ in the process; the result is a perfectly well-formed i–ii\textsuperscript{6/5}–V–i progression in minor. Example 8.4.c inverts the harmonies in example 8.4.a around middle C, producing a non-stylistic minor-key i–v\textsuperscript{♭}–iv–i progression. The contrast between examples 8.4.b and 8.4.c illustrates the general point that acceptable tonal progressions in major can usually be transformed into acceptable tonal progressions in minor by way of diatonic transposition, but only rarely by inversion.

We can conclude that Riemann’s inversionally invariant terminology, though theoretically elegant, is often inconsistent with the actual procedures of traditional tonal music.\textsuperscript{23} In this sense, it is a theory in search of a repertoire—a speculative description of what might be possible, rather than a faithful description of the music of Riemann’s time.\textsuperscript{24} It is rather remarkable that within a few years of Riemann’s death, Schoenberg had devised an entirely new musical language that fully embraced inversional equivalence—a language that teems with dualistically equivalent progressions, often resulting from the use of inversionally related twelve-tone rows.\textsuperscript{25} No doubt Riemann would have rejected Schoenberg’s music as violatingnatural laws of tonality. But from a more distant (and somewhat Whiggish) perspective, we can see twelve-tone music as a vindication of Riemann’s speculative music theory—albeit one that Riemann himself would have considered perverse.

3. Dualism and Voice Leading

The harmonic syntax of traditional tonal harmony does not exhibit even an approximate inversional symmetry, but what about the extended tonality of the late nineteenth century? Might it be the case that Riemannian dualism, while not useful for describing the diatonic “first practice” of nineteenth century music, can still tell us something about its chromatic “second practice”?

My answer is a qualified “yes.” Dualistic terminology is useful for analyzing chromatic tonality, and this helps explain why Riemann’s ideas have been so fruitful in recent music theory. However, I will suggest that contemporary theorists have
not produced a fully adequate account of why this is so. In my view, nineteenth-century composers were not explicitly concerned with inversional relationships as such; instead, these relationships appear as necessary by-products of a deeper and more fundamental concern with efficient voice leading. Rather than being the syntactic engine that drives the music, inversion is merely epiphenomenal—the smoke that escapes from the locomotive’s chimney, rather than the furnace that makes it go. And though dualism can be useful in analysis, this is largely because it is a tool that helps us to comprehend the range of voice-leading possibilities available to nineteenth-century composers.

Let me approach these issues by proposing a very simple model of late-nineteenth-century tonality, according to which the music combines a diatonic “first practice” inherited from eighteenth-century tonality with a chromatic “second practice” emphasizing efficient voice leading between familiar sonorities. This flexible “second practice” sets very few constraints on composers: virtually any voice leading between familiar chords may be used, as long as it is efficient. These chromatic voice leadings serve a variety of musical functions, acting as neighboring chords, passing chords, intensifications of dominants, modulatory shortcuts between distant keys, and so on. (Examples 8.5a–e provide a few representative passages from Schubert, Chopin, and Schumann.) Over the course of the century, one finds a gradual emancipation of the second practice, as chromatic voice leading—at first sporadic and decorative—controls ever-larger stretches of music.

The interesting point is that this concern for efficient voice leading will necessarily give rise to a wealth of dualistic relationships. For instance, example 8.6 lists the sixteen “semitonal” voice leadings between consonant triads (that is, voice leadings in which no voice moves by more than a semitone). The voice leadings have been grouped into inversionally related pairs. They have further been categorized by the retrograde relationship: the voice leading in column 3 is the transposed retrograde of the voice leading in column 1. (Note that the first and fourth columns are related nondualistically, with identical root motion connecting two major or two minor triads.) It is clear from the table that an interest in semitonal voice leading among consonant triads will necessarily give rise to dualistic relationships; indeed, even a composer who chooses

Ex. 8.5. Efficient voice leading in nineteenth-century music: (a) Schubert, D major piano sonata D. 850/Op. 53, I, mm. 11–16; (b) Schubert, Am Meer, m. 1; (c) Chopin, Nocturne Op. 9 no. 1, mm. 23–24; (d) Chopin, Nocturne Op. 9, no. 1, mm. 38–39; (e) Schumann, “Chopin,” from Carnival, mm. 11–13.
randomly from among the voice leadings in example 8.6 will generate numerous dualistically related progressions.

It turns out that the efficient voice leadings between members of any two set classes can always be grouped into inversionally related pairs. This is because transposition and inversion are (as discussed above) distance-preserving operations: thus, if a particular passage of music exhibits efficient voice leading, then we can invert the passage to obtain equally efficient voice leadings (cf. example 8.7, which uses inversion to categorize the voice leadings between half-diminished and dominant-seventh chords). Suppose, then, that a musical style obeys the following three principles:

(P1) If a sonority is acceptable then so is its inversion.
(P2) Efficient voice leadings are desirable.
(P3) Any additional voice-leading prohibitions—such as the prohibition on parallel perfect fifths—apply equally to ascending and descending motion.

In these styles, if a voice leading is acceptable, then its inversion will also be—which means we should expect a reasonable number of dualistic relations in the music itself.

Since principles P1–P3 accurately describe the nineteenth century’s “second practice,” dualism provides us with a useful tool for cataloging chromatic possibilities. Insofar as we want to develop a systematic grasp of all the efficient voice
leading possibilities between familiar sonorities—not just those in examples 8.6 and 8.7, but all the analogous voice leadings between all the familiar tonal chords—then our task will be simplified by cataloging voice leadings on the basis of inversive equivalence: without dualism, we would have to memorize each of the voice leadings in examples 8.6–8.7 separately; but once we understand that they are grouped into dualistically related pairs, we need memorize only half as many. However, dualism is just one of several tools needed here: both the retrograde relationship and what I have elsewhere called individual transpositional equivalence are also useful in this context. Together, these concepts allow us to reduce a very large set of voice-leading possibilities to a much smaller set of underlying paradigms.
These ideas derive, ultimately, from Richard Cohn, the first theorist to notice that dualistic terminology has a natural application to questions about voice leading. In “Maximally Smooth Cycles, Hexatonic Systems, and the Analysis of Late-Romantic Triadic Progressions,” Cohn asks, “which equal-tempered harmonies can be connected by single-semitone voice leading to at least two of their transpositions or inversions?” He answers this contrapuntal question dualistically, noting that two triads can be linked by single-semitone voice leading if and only if they are related by the neo-Riemannian L or P transformations. The present essay generalizes Cohn’s observation by observing that the efficient voice leadings between any two set classes can always be grouped into inversionally related pairs. Since inversionally related voice leadings move their voices by the same distances, dualism is a natural framework for investigating certain kinds of contrapuntal questions.

Following Cohn, then, I conclude that the chromatic voice leadings of the nineteenth-century’s “second practice” do exhibit an important kind of inversional symmetry: we can invert any stepwise (or semitonal, or efficient) chromatic voice leading between tertian sonorities to produce another such voice leading. Since chromatic music exploits virtually all of the efficient voice leadings between familiar chords, we should expect to find numerous dualistic relationships therein. For the same reason, we should expect to find retrograde relationships, as well as instances of individual transpositional equivalence. Inversion, retrograde, and individual transpositional equivalence are important primarily because they help us comprehend the range of options available to nineteenth-century composers—permitting us to group these voice leadings into categories whose members are related in interesting but nonobvious ways. Together, these categories bring a measure of order to the unruly world of nineteenth-century chromatic possibility.

4. Harmonic Dualism

Let’s contrast this “contrapuntal” dualism with a more orthodox form of dualism descending from Riemann himself. Harmonic dualists reject the suggestion that counterpoint produces inversional relationships, proposing instead that inversion is explanatorily basic. Thus the two dualisms have diametrically opposed understandings of the relative priority of harmony and counterpoint: one conceives of inversional relatedness as a tool for categorizing voice-leading possibilities, while the other understands inversional relationships as explanatory in their own right.

Consider, in this context, a brief but famous analysis by David Lewin. Lewin observed that the two Wagnerian passages shown in example 8.8 are interestingly related: example 8.8a, the Tannhelm motive, contains two minor triads (g♯, e) and an open fifth suggesting either b minor or B major. Example 8.8b, from the modulating section of the Valhalla motive, contains major triads on G♭, B♭, and F. The semitonal voice leadings at the beginning of each passage, (G♯, B, D♯) → (G, B, E) and (G♭, B♭,
D\(b\) → (F, B\(b\), D), are inversionally equivalent; hence from a dualistic perspective, they instantiate the same basic musical schema. \(^{32}\) The second progression in each passage, meanwhile, involves ascending-fifth motion, suggesting tonic → dominant (or subdominant → tonic) motion. Lewin thus constructs a single “transformational network” to describe the two passages: the first progression is described as “LP”—Riemann’s Leittonwechsel transformation followed by the neo-Riemannian “parallel” transformation, \(^{33}\) together producing Riemann’s Terzschritt; the second progression is labeled SUBD, indicating that the chord moves up by fifth.

The analysis, like a good deal of neo-Riemannian theory, is very much in the spirit of Riemann’s harmonic dualism. Lewin does not consider the idea that voice leading might help explain the first progressions in examples 8.8a and 8.8b. Nor does he differentiate the first progression in each passage, which arguably arises from efficient voice leading, from the second progression, which is a piece of traditional harmonic syntax. Instead, the “network analysis” in example 8.8d places a neo-Riemannian harmonic label (“LP”) alongside a more traditional harmonic label (“SUBD”). (The purely harmonic character of this network can be seen from the fact that it applies to any progression from G\# minor to E minor to B—even registrally disjunct passages such as example 8.8c.) The implication seems to be that the neo-Riemannian LP has a status akin to that of the traditional tonal I–V (or IV–I) progression. Since the harmonic routines of traditional tonality were clearly part of the cognitive framework of nineteenth-century composers, one might read Lewin as suggesting that dualistic harmonic ideas played a similarly important role. \(^{34}\) Certainly, he treats the inversional relationships as significant in themselves, rather than the mere by-products of deeper contrapuntal forces.

I am suspicious. From my point of view, analyzing Wagner while ignoring counterpoint is like trying to explain a locomotive’s motion on the basis of the shape of
the clouds emanating from the smokestack. This is, first, because the contrapuntal view explains facts that the harmonic view does not. From my perspective, it is not at all coincidental that the “Tarnhelm” and “Valhalla” motives exploit major third relationships between major and minor triads. Example 8.6 showed that two major triads can be connected by maximally efficient voice leading precisely when they are related by major third: voice leadings such as (G♭, B♭, D♭) → (F, B♭, D) move just two notes by one semitone, which is as small as any voice leading between major triads can be. (Inverting, we see that a similar fact holds true of E and G♯ minor triads.) This helps explain why we find so many triadic major-third relationships in pieces as different as Schubert’s G major string quartet (movement IV, starting measure 132), Wagner’s Ring, and the G minor prelude from Shostakovich’s op. 87 Preludes and Fugues. From this point of view, what is most striking about the chromatic progressions in example 8.8 is that they use maximally efficient voice leading between triads, not that they are related by inversion. By contrast, for the harmonic dualist, there is nothing particularly distinctive about major-third relationships: the focus of Lewin’s analysis is entirely on the relation between examples 8.8a and 8.8b, not on the individual contrapuntal qualities of each example considered in isolation—qualities that in my view help explain the relationship between the passages.35

Second, efficient voice leading potentially explains a wider range of Wagnerian procedures than does harmonic dualism. Example 8.9 presents a number of progressions drawn from Wagner’s Tristan, all using efficient chromatic voice leading between familiar tonal sonorities. Harmonic dualism offers no unified explanation of these progressions, nor of their relation to examples 8.8a and 8.8b. (After all, the mere fact that “LP” progressions appear in The Ring gives us no reason to expect that Wagner would elsewhere exploit semitonal voice leadings between seventh chords.) The contrapuntal view thus captures the intuition that there is a single compositional procedure that underlies a wide range of Wagnerian passages.36 Third, contrapuntal dualism offers a simpler and more elegant historical narrative. Composers and theorists have been concerned with efficient voice leading since the dawn of Western counterpoint. The contrapuntal dualist claims that nineteenth-century chromaticism is revolutionary chiefly insofar as it augments traditional tonal syntax with moments of efficient voice leading in chromatic space: thus, triads

Ex. 8.9. Chromatic Voice Leading in Tristan.
like E minor and G♯ minor, once thought to be tonally distant, came to be seen as close, since they could be connected by efficient chromatic voice leading. By contrast, it is harder to tell a plausible story that explains how nineteenth-century composers suddenly became attracted to dualistic harmonic procedures—particularly since there is so little historical evidence to support this suggestion.

These arguments, I suggest, pose a genuine dilemma for proponents of harmonic dualism. If a theorist believes dualistic transformations to be more than by-products of voice leading, then she will need to do more than simply point to sporadic instances of inversional relationships in nineteenth-century music. Instead, she will need to show that these relationships appear especially frequently, or play a significant musical role. If, on the other hand, the theorist does not want to undertake this project, then it is still incumbent upon her to produce some sort of metatheoretical justification for the emphasis on inversion. What is the point of singling out these particular harmonic relationships if we have good reason to think they are mere by-products? The danger is that a too-narrow focus will overemphasize the relations between examples 8.8a and 8.8b, and underemphasize the relations among the voice leadings in example 8.6. And this in turn may lead to an impoverished perspective on chromatic tonality.

5. **Brahms and the Tristan Chord**

How does my contrapuntal perspective contribute to analytical practice, if at all? Let’s explore this question by way of a brief analysis of Brahms’s Intermezzo, op. 76, no. 4—composed in 1878, thirteen years after the premiere of Tristan. Example 8.10a contains the opening phrase of the Intermezzo’s rounded binary form. Example 8.10b summarizes the contrasting middle section, while example 8.10c shows how the opening music is altered in the repeat. The typography reflects my claim that chromatic tonal music involves two distinct systems. Open noteheads represent chords that participate in the first-practice routines of functional tonality—each is assigned a Roman numeral indicating its harmonic function. Closed noteheads refer to chromatic chords whose function is largely contrapuntal. These have been assigned neither Roman numerals nor neo-Riemannian harmonic labels.

Brahms’s short piece exemplifies a relatively common nineteenth-century schema, systematically exploring the voice-leading possibilities of a few characteristic sonorities. Here, the relevant sonorities are the Tristan chord {F, G♯, B, E♭} and the E♭ minor triad. (Note that Brahms’s Tristan Chord is the actual Tristan chord, appearing at the correct pitch-class level.) The brief piece resolves the Tristan chord in three different ways: to F⁷ at α₁, to A♭ major at α₂, and to E♭ major at α₃. Similarly, the E♭ minor triad resolves to F⁷ at β₁, to G♭⁷ at β₂, and to B♭ major at β₃. This chromaticism tends to lead the music into distant keys: the second resolution of the Tristan chord, at α₂, takes us from B♭ major to A♭ major; the dramatic contrapuntal move from G
minor to Eb minor paves the way for a smooth transition into the Cb major of second phrase; and the return from Cb major to Bb major occurs by way of the Eb minor triad, here acting as iv of Bb major. Harmonically, then, efficient voice leading is a centrifugal force, pulling the music into new tonal territory. It is only at the end of the piece that this force is overcome, as the I–iv–I progression Bb → Eb → Bb tames Eb minor, returning it to the Bb major fold.

This technique is common in late nineteenth-century music; indeed, the Tristan prelude (and the opera as a whole) could be said to be “about” the various ways of resolving a Tristan chord to the dominant-seventh chord, while Chopin’s E-minor prelude can be said to be “about” the various ways of interpolating single-semitone
voice leading into a descending-fifth sequence of seventh chords. Brahms’s piece, like these others, illustrates the nineteenth-century principle that any chord can move to virtually any other chord, so long as the two can be connected by efficient chromatic voice leading. But where this description might suggest a kind of unregulated chaos, Brahms is characteristically disciplined: rather than populating the piece willy-nilly with unrelated examples of chromaticism, he returns repeatedly to a few sonorities, demonstrating their capabilities rather like a traveling salesman exhibiting the many functions of an expensive vacuum cleaner.

Indeed, there are at least four ways in which Brahms ensures the Intermezzo’s coherence. First, the opening phrase features a clear stepwise ascent from F to D, shown by the stems in example 8.10a; in the return, the rising stepwise line is balanced by a chromatic linear descent from E♭₃ to B♭₂ (example 8.10c). Second, the piece is suffused with Brahmsian motivic connections, particularly the double-neighbor B₄ – D₅ – C₅ from the second measure. Third, as noted above, the piece returns repeatedly to the same small set of sonorities: not just the Tristan and E minor chords, but also G♭⁷, which appears both the middle section of the piece and at γ₃ as a neighbor to B♭. Finally, though the piece is reasonably chromatic, Brahms never lets these centrifugal forces overwhelm the diatonic elements: the music clearly articulates numerous points of tonal arrival and often allows the listener to track the play of diatonic functions. Together, these four factors moderate the “anything goes” radicalism of chromaticism, producing a delicious Brahmsian blend of extravagance and restraint.

These observations suggest the more general thought that coherence in nineteenth-century music is to be found at the level of specific musical works, and not at the level of general syntactical principles. It is, I think, indisputable that chromatic harmony permits virtually any efficient voice leading between familiar chords. But to say this is not to deny that it takes compositional skill to deploy these options in a musically satisfying way—on the contrary, the more possibilities available to a composer, the harder it is to build logical musical structures. To understand how nineteenth-century composers constructed intelligible pieces, one must therefore look closely at individual works: it is here, and not at the level of universal laws of chromatic tonal syntax, that interesting constraints on musical coherence are to be found. I suspect that careful analysis of nearly any successful nineteenth-century music would reveal interesting strategies for harnessing the inherently destabilizing force of chromatic voice leading—techniques that prevent contrapuntal liberty from devolving into musical anarchy.

In section 3, I suggested that we cannot develop a true understanding of chromatic tonality unless we have a systematic mastery of the voice-leading possibilities between chords. Once we have developed such mastery we will see that Brahms’s piece is not a series of idiosyncratic contrapuntal gestures, but a collection of very familiar moves. For example, the voice leading at α₁ appears at measures 97–98 of the Tristan prelude (example 8.9e); the voice leading at γ₁ appears in example 8.8a; the voice leading at γ₂, which features ♯II⁷ acting as a tritone substitution for the F⁷ chord, is very similar to the first voice leading in Tristan (example 8.9a); the voice leading at α₃ is very similar to the final voice leading in Tristan (example 8.9g); the voice lead-
ing at \( \gamma \) opens Schubert’s song “Am Meer” (example 8.5b); and so on.\(^3\) The piece’s various chromatic moves are no more original to Brahms than are its elements of traditional tonal syntax. Indeed, in my view, the efficient chromatic voice leadings constitute the shared syntax of the nineteenth-century’s second practice, just as the shared routines of eighteenth-century tonality constitute its first practice. To understand voice leading is to understand the space in which nineteenth-century composers operated—and is in turn prerequisite for appreciating the often astonishing skill with which they deployed the opportunities available to them.

6. Conclusion

The idea of symmetry thus provides a unifying thread that runs throughout the history of music theory, from Rameau to Weber to Riemann. We could in principle follow this thread into the twentieth century, for example by interpreting Schoenberg as eliminating one of the traditional symmetries: where traditional theorists sometimes consider the order of a group of pitches to be relatively unimportant, twelve-tone rows are defined by their order. Appreciating Schoenberg’s music thus involves a two-stage process: not only do we need to sensitize ourselves to the orderings of twelve-tone rows; we also have to desensitize ourselves to their unordered pitch content—since from this point of view, twelve-tone rows are all the same. (Indeed, if one looks at the pitch-class content of moderate spans of music, twelve-tone pieces are remarkably homogenous: rather than modulating from one scale to another, they continually recirculate through the same twelve pitch classes, creating a kind of middle-ground harmonic uniformity.) Whether this twofold reorientation is psychologically possible or aesthetically desirable is a complex and fascinating question. Unfortunately, a thorough discussion of these ideas is a matter for another time.

Instead, let us conclude by reconsidering the ambiguous role of inversional symmetry in tonal music. It is clear that inversion is, in some sense, a genuine symmetry of the musical universe. Since inversion and transposition are the only distance-preserving operations on pitch space, there are numerous situations in which it can be useful to think dualistically. But while inversion and transposition are equally important mathematically, they are not equally salient psychologically. Many tonal styles take advantage of transpositional symmetry, permitting characteristic musical patterns to appear at virtually any pitch level. Individual pieces are often performed in multiple keys, for instance, to accommodate different vocal ranges. By contrast, it is hard to think of a robustly tonal style that wholeheartedly embraces inversional symmetry.

Nevertheless, it is clear that inversional relationships occur reasonably frequently in chromatic tonal music. Consequently, dualist language can help us describe genuine relations present in this repertoire—witness Lewin’s interesting comparison of examples 8.8a and 8.8b. At the same time, however, it is possible to overemphasize the
significance of these relationships, since there is good reason to think that they often arise as the by-product of a concern for voice leading. Indeed, it seems likely that a composer or analyst could become expert in chromatic tonality without any explicit awareness of inversional symmetry: one would simply have to learn the voice leadings in examples 8.6 and 8.7 (as well as many other analogous voice leadings between other familiar chords) individually, rather than as inversionally related pairs. By contrast, it would be impossible to become an expert tonal composer without understanding transposition in a general and systematic manner.

Analyses of nineteenth-century music therefore need to walk a fine line, exploiting dualism for what it can give us, while being careful not to overestimate its role in the music itself. I have suggested that the prudent approach is to interpret inversion—like retrograde and individual transpositional invariance—as a tool we use to organize and catalog the musical possibilities available to nineteenth-century composers. To do this is not to deny outright the importance of dualistic theorizing, but it is to reframe its significance somewhat, requiring that analysts adopt a somewhat circumspect attitude toward its claims. For in the language of another great dualist, it is possible that inversional symmetry is a feature of chromaticism as it appears to us, not as it is in itself.

NOTES

Thanks to Scott Burnham, Elisabeth Camp, Noam Elkies, Ed Gollin, Dan Harrison, Alex Rehding, and Robert Wason for their help with this article.


2. It should be emphasized that this conception of symmetry inheres in our basic notation and terminology; we are not talking about the manifest symmetry of, say, a palindromic piece.

3. The essays in William Kinderman and Harald Krebs, eds., The Second Practice of Nineteenth Century Tonality (Lincoln: University of Nebraska Press, 1996), explore the idea that the nineteenth century, like the early seventeenth century, had a "first practice" and a "second practice." I return to this thought in section 3.


5. I use scientific (i.e., Acoustical Society of America) pitch notation in which middle C is C₄; spelling is unimportant. Regular parentheses ( ) denote ordered lists, while curly braces { } denote unordered collections.

6. Of course, we intuitively consider (C₄, E₄, G₄) to be very similar to (E₄, C₄, G₄), since they are related by reordering. This shows that we instinctively adopt certain musical symmetries even without realizing it.

7. Various theorists prefigured Rameau with respect to triads. However, Joel Lester credits Rameau with asserting a more general principle that applies to seventh chords as

8. For example, transposition transforms the C major chord (E₂, C₅, G₄, G₃) into the D major chord (F♯₂, D₅, A₄, A₃). Note that we can use the octave symmetry to shift just some of the notes in an object—for instance, transforming (C₄, E₄, G₄) into (C₅, E₄, G₄)—whereas we must apply the same transposition to all the notes in an object.

9. It should be noted that this symmetry is only approximate, since second-inversion triads have an anomalous status in tonal harmony.

10. To transform the first chord of example 8.1a into the first chord of example 8.1b, one needs to switch the notes played by soprano and alto, and then transpose the soprano voice up an octave. However, this operation will not transform the second chord of example 8.1a into the second chord of example 8.1b.

11. Here we transpose the first chord in example 8.1c down by four semitones, the second down by seven, the third down by three, and the fourth down by seven.

12. Triadic extension symmetry represents a slight departure from Rameau’s ideas: Rameau viewed the chord {D, F, A, C} as both being a D chord with added seventh and an F chord with added sixth. By contrast, the triadic extension principle is typically associated with the view that all harmonies are fundamentally “stacks of thirds.”

13. Chromatic transposition shifts notes by a constant number of semitones; diatonic transposition shifts notes by a constant number of scale steps. Because of this, one may have to change capital Roman numerals to small Roman numerals, and add accidentals to raise the leading tone; I will ignore these details here.

14. Ian Quinn proposes replacing the standard Roman numeral/figured bass system with an alternative, quasi-Riemannian, system that explicitly represents harmonic functions. See his “Harmonic Function without Primary Triads” (paper presented to the national meeting of the Society for Music Theory in Boston, 2005). As far as I can tell, Quinn’s system makes it difficult to express principles like “IV goes to ii but not vice versa” or “roots rarely progress by ascending third.”


16. A subtle point: it is perhaps more accurate to view Riemann as attempting to relate major keys and minor keys by inversion rather than diatonic transposition. If so, then it is inaccurate to say he wanted to extend the Rameau/Weber symmetry group—instead, he wanted to change it by replacing one symmetry (diatonic transposition) with another (inversion). This subtlety will not be relevant to the following discussion.


18. The inversion of any tertian sonority is also a tertian sonority. Hence, when we list all the tertian sonorities of a given cardinality, we find they can be grouped into inversionally related pairs. (Some, of course, are their own inversions.)

19. Suppose we have a chord progression A → B with n common tones. Inversion can be used to produce a second progression, Iₙ(A) → Iₙ(B), between sets of the same set class, which also has n common tones. This was well known to Riemann, and is discussed in David Kopp’s Chromatic Transformations in Nineteenth-Century Music (Cambridge: Cambridge University Press, 2002), 74.

21. Here I deviate from David Lewin, who interprets Riemann’s *Schritte* and *Wechsel* as “transformations” or *functions* that, upon being given a chord as input, return some other chord as output. See his *Generalized Musical Intervals and Transformations* (New York: Oxford University Press, 2007), and “Some Notes on Analyzing Wagner: ‘The Ring’ and ‘Parsifal,’” *19th-Century Music* 16.1 (1992): 49–58. I have instead treated chord progressions as higher order musical objects related by transposition and inversion. My approach, unlike Lewin’s, permits “dualistic” progressions between chords with different symmetries (for example, C⁷→C°⁷ and C⁷→C♯⁷). No (single-valued) function over chords can capture this sense of “dualistic equivalence,” since it would be necessary to map a single augmented chord to multiple diminished sevenths.


23. These inadequacies may have motivated Riemann’s eventual introduction of nondualistic “functional” harmonic labels—a second theoretical system that coexists only somewhat uncomfortably with Riemann’s dualism (see Kopp, *Chromatic Transformations*, and Rehding, *Hugo Riemann*). However, even function theory does not smoothly account for the diatonic transpositional symmetry between major and minor: Riemann would label the major-mode submediant (vi) as “Tp,” while labeling the minor-mode submediant (VI) as “F”—even though the submedian triad behaves similarly in the two modes.

24. Rehding, *Hugo Riemann*, chapter 3, has cautioned that it is somewhat anachronistic to read Riemann as if he were a contemporary theorist, concerned only with describing the behavior of actual composers; instead, he is (at least in part) a speculative theorist who aimed to provide directions for future compositional work.

25. Consider, for example, the opening of Schoenberg’s op. 33a. If one were to try to analyze the piece without twelve-tone terminology, one might emphasize the retrograded dualistic relationship between the second two chords in the first measure and the first two chords in the second measure. See David W. Bernstein, “Symmetry and Symmetrical Inversion in Turn-of-the-Century Theory and Practice,” in *Music Theory and the Exploration of the Past*, ed. Christopher Hatch and David W. Bernstein (Chicago: University of Chicago Press, 1993), 377–409, for a more general discussion of symmetry and Schoenberg.

26. By “efficient” voice leading I mean, roughly, “voice leading in which no voice moves very far.” See my “Voice Leadings as Generalized Key Signatures,” *Music Theory


28. I begin each voice leading from C major or C minor. Voices are individuated by register: the top note in the first chord moves to the top note in the second; the middle note in the first chord moves to the middle note in the second; and so on. Intuitively, a “voice leading” corresponds to a phrase like “C major moves to E major by holding E constant, moving C down by semitone to B, and G up by semitone to G♯” (see A Geometry of Music or “Scale Theory, Serial Theory, and Voice Leading”). Mathematically, voice leadings are equivalence classes of progressions under (uniform) applications of the reordering and octave-shift symmetries (see Callender, Quinn, and Tymoczko, “Generalized Voice Leading Spaces”).

29. As the name suggests, individual transpositional equivalence results from treating transposition as an individual, rather than uniform symmetry. Consider the semitonal voice leading \((C_4, E_4, G_4) \rightarrow (C_4, E_3, G_3)\), shown in the third staff of example 8.6. (The notation \((C_4, E_4, G_4) \rightarrow (C_4, E_3, G_3)\) indicates that \(C_4\) moves to \(C_4\), \(E_4\) to \(E_3\), and \(G_4\) to \(G_3\).) By transposing the destination sonority up by semitone, we can obtain a closely related voice leading, \((C_4, E_3, G_3) \rightarrow (C_3, E_2, G_2)\), shown on the fourth staff. These two voice leadings each map root to root, third to third, and fifth to fifth, moving their voices by the same distances, up to an additive constant. For more discussion of this idea, see A Geometry of Music and “Scale Theory, Serial Theory, and Voice Leading.”

30. Some of Cohn’s writing suggests a more harmonic understanding dualism, of the type we will consider in section 4. For instance, in “As Wonderful as Star Clusters: Instruments for Gazing at Tonality in Schubert,” Nineteenth-Century Music 22.3 (1999): 213–232, Cohn seems to portray hexatonic key areas as syntactically significant harmonic regions, rather than mere by-products of semitonal motion. It would be interesting to trace the interacting themes of harmony and counterpoint in Cohn’s work.

31. Riemann has often been criticized for ignoring counterpoint. See, for example, Milton Babbitt, Words about Music (Madison: University of Wisconsin Press, 1987), 136–137.

32. Recall that the notation \((G♯, B, D♯) \rightarrow (G, B, E)\) indicates that \(G♯\) moves to \(G\), \(B\) moves to \(B\), and \(D♯\) moves to \(E\). Inverting the voice leading \((G♯, B, D♯) \rightarrow (G, B, E)\) gives us \((G, B♯, D♯) \rightarrow (F, B♭, D)\). For more, see my “Voice Leadings as Generalized Key Signatures,” “The Geometry of Musical Chords,” “Scale Theory, Serial Theory, and Voice Leading,” and A Geometry of Music.

33. The neo-Riemannian parallel transformation is equivalent to Riemann’s Quintwechsel, transforming C major into C minor.

34. This conjecture is somewhat speculative, since Lewin does not provide much discussion of the significance of the inversional relationship. I am operating on the assumption that Lewin thought the relationship was important (since he wrote a paper
about it) and that he thought it was more than a by-product of contrapuntal forces (since otherwise he would have mentioned voice leading).

35. It should be noted that any instance of maximally efficient voice leading between two major triads can be related by inversion and (possibly) retrograde to any instance of maximally efficient voice leading between two minor triads—or, to put it another way, the two progressions can be described as exemplifying a single "transformation" drawn from the set \{LP, PL, T_4, T_8\}. In the grand scheme of things, then, it is not all that surprising that a single network describes examples 8.8a–b. We would expect the Ring's fifteen hours of music to contain several instances of maximally efficient voice leading between two major or minor triads, followed by an ascending fifth progression; and from these passages we would expect to be able to select pairs that exemplify a single Lewin-style network.

36. Note also that a contrapuntal approach predicts that triads and seventh chords should in general progress in different ways, since they have different voice-leading capabilities. (In particular, seventh chords are particularly close to their minor-third and tritone transpositions, just as triads are particularly close to their major-third transpositions.) But a purely harmonic approach would give us no reason to expect systematic differences between chords of different sizes. Thus we could potentially use statistical analysis to adjudicate between these two explanations.

37. Lewin explores Brahms’s frequent references to the formal and rhetorical procedures of earlier eras. See his “Brahms, His Past, and Modes of Music Theory” in Brahms Studies: Analytical and Historical Perspectives, ed. George S. Bozarth (Oxford: Oxford University Press, 1990). The appearance of the Tristan chord suggests that Brahms may sometimes have made similar reference to the work of his contemporaries.

38. Some readers may prefer to analyze the chord at \(\alpha_1\) as a diminished seventh chord \(\{B, D, F, A^b\}\) with an \(E^b\) pedal, or as the ninth chord \(\{D, F, A^b, B, E^b\}\). The important point is that the same sonority occurs at \(\alpha_1\), \(\alpha_2\), and \(\alpha_3\).

39. Note that the main key areas of \(B^b\) major and \(C^b\) major are largely expressed by their dominants, with the tonic \(B^b\) major chord not arriving until the very end of the piece. This chromatic practice is analyzed at length in Robert Morgan’s “Dissonant Prolongation: Theoretical and Compositional Precedents,” Journal of Music Theory 20 (1976): 49–91. The large-scale semitonal key relation is also common; see Patrick McCreless, “An Evolutionary Perspective on Nineteenth-Century Semitonal Relations,” in The Second Practice of Nineteenth Century Tonality, eds. William Kinderman and Harald Krebs (Lincoln: University of Nebraska Press, 1996).

40. The Tristan prelude is of course also about longing and death. I am engaging in composer’s shoptalk here, speaking about how the notes are assembled, rather than expressive use to which they are put.

41. Also notable in this regard are the overlapping lines in the contrasting middle section. These motivic details are not shown on the harmonic reduction.

42. Following analytical tradition, I am discarding Tristan’s motivic voice exchanges. See A Geometry of Music.

43. The main pivot-chord modulation in the piece, in which iv of \(B^b\) major becomes iii of \(C^b\) major, is related to the first modulation in Schubert’s “Die junge Nonne,” where VI of \(F\) minor becomes V of \(F^\#\) minor. The harmonic minor scale has two major triads a semitone apart, whereas its inversion, the harmonic major scale, has two minor triads a semitone apart. Schubert exploits the former property and Brahms the latter, with their two modulations being dualistically equivalent.