

OUCH THEORY

Josephus. Oooooouccccchhhh

Aloysius. What's wrong, young Josephus? You don't seem yourself today.

Josephus. I come to you, venerable master, with the worst headache I've had in years.

Aloysius. Is it homecoming week already? It seems a little early in the semester for that ...

Joseph. No, noble master, far from it. Actually, I really wish I could have some old-fashioned undergraduate fun! Instead, my head is throbbing from our last meeting.

Aloysius. But we stopped hitting students ages ago, once tuition went so high. Or did I slip up again?

Josephus. Hardly, gentle master. Instead, you gave me an endless list of seemingly arbitrary musical rules and told me to go home and try to follow them.

Aloysius. So what? Since time immemorial, students have proved themselves by avoiding parallel fifths and octaves, incomplete chords, awkward leaps, melodic tritones, and the like. This is how we separate the sheep from the unwashed hordes with their lutes and electric guitars.

Joseph. But, noble master, the trouble is that you have given me no advice about what I *should* do. I stare at my staff paper but nothing comes to mind.

Aloysius. Oh, that's simple! Everything is permitted as long as it is not prohibited!

Joseph. But don't you see the problem? You've dropped me into a massively multidimensional fitness landscape without telling me how to find the peaks!

Aloysius. Come again?

Joseph. Sorry. What I meant is that there are so many rules that it is very difficult to follow them all. I can barely keep them in my head at once! As a result, writing music has become a frustrating process of trial and error: I imagine a possible progression, consult my notes, and realize after several minutes that it violates one of about a thousand different rules. Then it's back to the drawing board for me.

Aloysius. That's funny, because I seem to be able to write counterpoint quickly and without much trouble ...

Josephus. Well I feel like I'm living in a musical police state where the criminal code is too complex to understand! I decide to go to the movies, and then find out that movies are prohibited on Tuesdays after you've had broccoli for dinner!

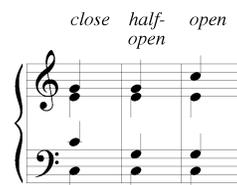
Aloysius. Hmm.

Josephus. (texting girlfriend) "trapped in office hours back ASAP"

Aloysius. (long pause) Well, it seems that there *must* be some kind of knowledge that allows me to write good counterpoint without thinking too much ... Perhaps it would help if I tried to describe some of the tricks I seem to use when I write?

Josephus. (skeptically, still holding his head) Will it make the throbbing stop?

Aloysius. (getting excited) For instance, in four-part counterpoint, it seems to me that the upper voices are typically found in one of three basic configurations: *close position*, where they sound a complete triad and are as close together as possible (e.g. C4-E4-G4), *half-open position*, where tenor and soprano are an octave apart with alto sounding a chord tone between them (e.g. G3-E4-G4), and *open position* where there is one unsounded chord tone between each of the three upper voices (e.g. G3-E4-C5, where chord tones C4 and G4 are not sounded). The three positions are ordered, with the voices closest in "close" position, somewhat farther apart in half-open position, and farthest apart in "open" position.



Josephus. I've heard of "close" and "open," V.M., but not "half-open."

Aloysius. Yes, I just made that one up—as far as I know there's no commonly accepted way to describe it.

Josephus. You said that the voices are "typically" found in these positions. What is that supposed to mean, exactly?

Aloysius. That these three patterns account for just over 85% of the upper-voice configurations of the triads in the Bach chorales.

Josephus. Bach, again? What about Goudimel?

Aloysius. Actually, the figure is *much* higher for Goudimel, but let's forget him for now. Learn to write like Bach and your headache will disappear, I promise you.

Josephus. Hmph. OK, so how does it help to know about these configurations?

Aloysius. It turns out there are some basic principles governing their relations—a kind of “grammar” of upper voice configurations. For instance:

1. Close position is most common, followed by open position, followed by half-open.
2. Close position tends to go to close position.
3. Open position tends to go to open position.
4. Half-open almost never goes to half-open.

Josephus. Wait, slow down! You’re giving me rules again!

Aloysius. Yes, but notice that these are norms or guidelines about what you *can* do, rather than rules about what to avoid. To go back to your police-state analogy—which, by the way, was uncomfortably close to the mark—it’s like knowing that you definitely *are* aloud to read comic books on Tuesday, no matter what you’ve had for dinner.

I. The “3+1 trick”

Josephus. Well that’s something I guess. Can you explain a little more?

Aloysius. Let’s start with the simplest, most important case, which is when one close-position configuration moves to another. I call this the “3+1 trick.” This should be your basic approach to four-part harmonization on homework and in exams.

The fundamental principle is that if the upper three voices, *considered on their own*, are sounding a root position triad, then they cannot move to another root-position triad. (If they do, you have parallel fifths.) As long as you obey this rule, the upper voices will never produce forbidden parallels. This means you only have to check for parallels with the bass. And it turns out that this is relatively easy to do.

By the way, we can use the term “upper voice formation” to refer to the inversion that the upper voices are in—the inversion the chord would have if the bass were absent.

This schema is *very useful* for time-limited situations such as exams. In fact, you’re pretty much doomed if you don’t manage to internalize it. However, with a little practice, you should be able to use this technique to harmonize melodies and bass lines almost instantaneously. And I really do mean instantaneously, since people do manage to sight-read figured-bass parts.

how to think about it: *how to write it:*

The musical notation consists of two systems. The first system, labeled "how to think about it:", shows a treble clef with a G major triad in close position (G-B-D) and a bass clef with a G bass line (G-A-B-A). The second system, labeled "how to write it:", shows the same triad in open position (B-D-G) with the bass line moving to a half-note G, and the upper voices moving in parallel motion: G4 to A4, B4 to C5, and D4 to E4.

Josephus. But isn’t that cheating? I was told that four-part harmony was all about constructing four totally independent melodies! Here it seems like the upper voices are all versions of the same basic melodic pattern ...

Aloysius. You have to crawl before you can walk. Worry about beautiful lines once you are able to make complete triads and avoid forbidden parallels! But it's interesting that Bach's chorales—widely agreed to be beautiful examples of four-part triadic writing—make heavy use of this “3+1 trick.” This proves that you can write great counterpoint even while still using the trick all the time!

Josephus. OK, but what about parallels with the bass?

Aloysius. Now it pays to think systematically. Below, I've shown diatonic progressions by ascending second, third, and fourth. I start each progression from G, but you can transpose diatonically to get any other diatonic progression. You can also get descending progressions, or progressions by ascending fifth, sixth, and seventh by “retrograding” those on the example (i.e. playing them backwards). And of course, you can rearrange the registral position of the upper voices. This means that every possible diatonic progression can be derived from these basic possibilities.



In each case, I list the two most common upper-voice instances of the “3+1 trick.” (That is: the two ways of moving the upper voices that involve the smallest motion in the voices, one ascending and the other descending.) In the bottom staff, I list forbidden bass lines for each progression. Since second-inversion triads are generally prohibited, there are at most four possible bass lines for each case: root→root, root→third, third→root, and third→third.

Now notice that *every forbidden bass line creates parallel octaves with the upper voices!* This means you can forget all about fifths and focus only on octaves—which, by the way, are a little easier for beginning students to identify. In fact, if you sit at the piano and put the upper voices in the right hand, with the bass in the left, the forbidden bass lines will feel wrong, because you can tell that your two hands moving in parallel. By the same token, if your two hands move in contrary motion then you know you are OK.

Think about it this way. When writing four-voice counterpoint, you have to check for parallel fifths and parallel octaves between 6 pairs of voices, giving you a total of twelve tests to perform. When you use the “3+1 trick” you can cut this down considerably. You first check to make sure your upper voices are not moving from root position to root position. You then check whether your bass is in contrary motion with the upper voices. If so, you're OK. If not, check for parallel octaves between the bass and the voice that is an octave it. This can be done very quickly, even if you don't want to memorize the specific bass-voice prohibitions I've shown you.

Josephus. Hey, that's cool ... but aren't you ignoring some rules? My friend told me she learned that you shouldn't double the third of a first inversion triad—meaning that if you have a complete triad in the soprano, you can't have the third in the bass.

Aloysius. A common misconception. There's absolutely nothing wrong with doubling the third of a first-inversion triad: more than 1/3 of Bach's first-inversion triads have a doubled bass. Palestrina¹ too.

Aloysius. So does the "3+1 trick" work for open-position chords as well? It seems like there might be some new problems here.

Josephus. Very true! With open chords, there are two disallowed sequences of upper-voice formations: root position to root position and first inversion to first inversion. Thus with open position you're mostly going to want to change inversions from chord to chord. (Remember that I'm talking about the upper-voice formation, or the upper voices *considered in isolation from the bass.*)

the same formula in "open" position:



About 10% of the triadic voice leadings in the Bach chorales are of this form, meaning that fully 40% of the triadic voice leadings use the "3+1 trick" to either connect close to close or open to open. That's almost half!

Note that there is a useful trick for harmonizing in open position: write your harmonization in close position, and then transpose the alto voice down by octave, making it the new tenor (and making the old tenor into the new alto). For instance, if you look at the above example, you can transpose the tenor up an octave to create a close-position harmonization.

Josephus. It also seems like it should be possible to move between open and close positions.

Aloysius. In fact, about 6% of the triadic voice leadings in the Bach chorales do so. When changing between positions you can be sure that there won't be parallels in the upper voices, so you only need to check for fifths and octaves with the bass. **Actually, these sorts of change of position are the normal way to harmonize large leaps in the melody.**



¹ Giovanni Pierluigi da Palestrina was as you know the greatest master of late-Renaissance counterpoint. The "luigi" part of "Pierluigi" translates to "Aloysius" in Latin. The great theorist Johann Joseph Fux^a wrote his great textbook, *Gradus ad Parnassum*,^b as a dialogue between Josephus (= Fux himself) and Aloysius.

^a pronounced "Fooks." Google with care.

^b roughly, "steps to heaven"

Josephus. (thinking a bit) This all seemed a little magical when you started talking about it, but I'm starting to catch on ... in both open and close position, the upper voices always sound three different notes, meaning they will never form parallel octaves. So the only potential problem is fifths. When the voices are in close position the fifths can occur only between tenor and soprano, and only when the upper three voices are in root position. Whereas with open position chords, the fifths occur between *adjacent voices*, and can appear in both root position (between tenor and alto) and first inversion (between alto and soprano). So with close position triads, you can't move root position to root position, and with open position triads you can't move root position to root position or first inversion to first inversion. This also why there will never be fifths between upper voices when moving between open and close.

Meanwhile, the bass can only create parallel fifths with an upper voice if it moves from root to root, while the upper voice moves from fifth to fifth. But since we're using the "3+1 trick," an upper voice will move from fifth to fifth only if another upper voice moves from root to root, making parallel octaves with the bass. So as long as we are using the "3+1 trick" we can forget about fifths between the bass and upper voices!

II. Half-open chords.

Aloysius. Very good Josephus, you are catching on! Ready to think about half-open chords?

Josephus is text messaging on his cellphone, and does not hear the question.

Aloysius. Josephus?

Josephus. Wha-? Oh, um, sorry. Will they be on the exam?

Aloysius. You know, I'm getting the feeling that they might.

Josephus. Well let's keep going then ... (*brightly*) you seem awfully interested in this stuff, and it would be a shame to hold you back!

Aloysius (sighing). In half-open position, soprano and tenor are an octave apart and the upper voices, on their own, sound an incomplete triad. The bass voice adds the missing note. For instance, the upper voices might have C4-F4-C5 while the bass adds A3, completing the F major triad. One half-open voicing almost never moves to another half-open voicing, as this typically creates parallel octaves between tenor and soprano; hence, you usually move the half-open configuration to either open position or close position.

The image contains two musical staves. The first staff, labeled "think:", shows a soprano line with notes C4, F4, C5 and a bass line with note A3, forming an F major triad. The second staff, labeled "write:", shows a soprano line with notes C4, F4, C5 and a bass line with notes A3, C4, F4, forming an F major triad in first inversion.

Josephus. Except when the tenor and soprano are held fixed: then you can move from one half-open position to another without creating parallels ...

Aloysius. This is true, but it is very rare. Because of the two held voices, such progressions work better in eighth notes than in quarter notes, so we will put them aside for now. Focus instead on the default case, in which half-open moves either to close or open. Half-open to close (or vice versa) will *never* create parallels in the upper voices. However, half-open to open (or vice versa) can occasionally create fifths: to check for these, look for a pair of adjacent voices sounding a perfect fifth in the half-open chord; these should not progress to a perfect fifth in the open chord.

Josephus. What about parallel fifths and octaves with the bass?

Aloysius. If there is a half-open chord in your progression, you can never have parallel octaves, since there is no octave above the bass of a half-open chord. (Remember, in half-open position the upper voices form an incomplete triad with the bass completing the chord.) So you only need to check for fifths above the bass note, which will only happen when the bass moves from root to root.

Josephus. So, like, what's the point of the half-open chords?

Aloysius. I'm sorry, what?

Josephus. Is it that the half-open chords act as intermediaries, smoothing the transition between close and open? Or do you use them in a neighboring way, like when close moves to half-open and then back to close?

Aloysius. Josephus, that is very good, they do both of those things. Here, have a piece of chocolate, maybe it will help your headache. And while you're eating I will mention that about 27% of the triadic voice leadings in the Bach chorales connect half-open voicings to either close or open voicings.

Josephus. Boy, you really like numbers, huh?

Aloysius. Only because they provide a sense of which moves are most common and which are rare. When I get an exercise in which the chords are constantly bouncing back and forth between open and close positions, I can tell that the student doesn't really know what they're doing—even if they haven't violated any rules. And when I see something that's entirely close position, I think the student is a beginner, just starting to figure things out. On the other hand, a judicious use of half-open chords makes me start to reach into my jar of "A" grades. These harmonizations just "look right"—meaning they have the general feel of something written by a classical composer.

Josephus. Not that anyone here cares about grades, exalted M! We have our eyes on the higher things! Which reminds me, can you write six letters of recommendation for medical school, all due tomorrow?

Aloysius. That will depend on how quickly you absorb the following table, which summarizes our discussion so far. A “-” in a square means you will not create upper-voice parallels. In the other squares I’ve noted what you need to watch out for.

	CLOSE	HALF-OPEN	OPEN
CLOSE	no $\frac{3}{3} \rightarrow \frac{3}{3}$	–	–
HALF-OPEN	–	<i>forbidden unless soprano and tenor are still</i>	<i>look for perfect fifths between adjacent voices</i>
OPEN	–	<i>look for perfect fifths between adjacent voices</i>	no $\frac{5}{3} \rightarrow \frac{5}{3}$ no $^6 \rightarrow ^6$

Remember to check for parallels between upper and lower voices: check only for octaves when using the “3+1 trick” and check only for fifths when using a half-open chord!

III. OUCH theory

Josephus. (yawns)

Aloysius. It is possible to do a perfectly adequate four-part composition using only open, half-open, and close-position configurations. But to write something really excellent, you may need to incorporate the “unusual” voicings that belong to none of these categories. This gives four possibilities for the upper three voices, which form the handy acronym OUCH ... Open, Unusual, Close, and Half-open.

Josephus’s eyes are drooping. Aloysius, having surreptitiously grabbed a yardstick, now gives Josephus a sharp whack on the head.

Josephus. OUCH! ... oh haha, very funny.

Aloysius. You know, my jokes worked a little better back when I was noticeably younger than the old-timers who’d been here forever.

Josephus. Why are you telling me this, old tim—I mean, venerable master?

Aloysius. Never mind. Anyway, roughly 15% of the triadic voicings in a Bach chorale will be “unusual.” These voicings are dominated by two subcategories: *doubled interval*, in which two upper voices sound the very same pitch, with the remaining voice less than an octave away, and “octave plus” voicings with a full octave between either tenor and alto or alto and soprano. Like the half-open voicings, these unusual configurations are incomplete, with the bass providing the missing note. In fact, you can think of them as variants of the half-open configuration: doubled interval is like half-open where the top note has been moved down by octave (or the bottom note up by octave); octave-plus voicings are like half-open voicings in which the middle note has been moved up or down by octave.

We now have five basic possibilities, which can be ordered by the distance between the upper voices.

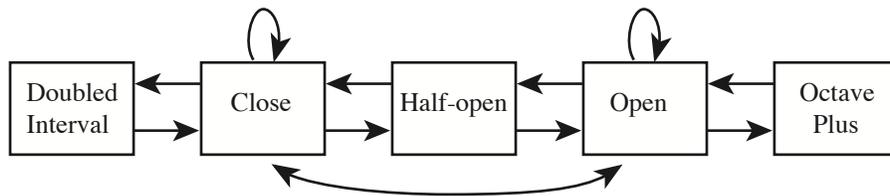


Josephus. Let me guess how this works. Since counterpoint generally involves voices moving by short distances, that means you don't want to move too far along this chain: the doubled interval configuration should generally go to the close configuration; close should generally go to doubled interval or half-open; half-open should generally go to close or open; open goes to half-open or octave plus. Moving from doubled interval to open, for example, will create simultaneous leaps in all voices and be awkward.

Aloysius. Josephus, you are really getting it. Perhaps someday you will pass this knowledge on to your own students!

Josephus. Or maybe a good grade in this class will pass me on to a job at Goldman Sachs?

Aloysius. (brightly) Maybe! In any case, as a practical matter, you won't go too far wrong if your basic choices are guided by the following map.



This provides a basic, first-approximation “grammar” of upper-voice configurations in four-voice triadic harmony, accounting for about 85% of the triadic voice leadings in Bach’s chorales. You can write a lot of decent music using this map.

Now, when moving between the doubled interval and close, you have to check for parallel fifths between soprano and tenor. (These occur only if the close triad is in root position and if the doubled interval formation is a perfect fifth.) When moving between the octave-plus and open configurations, you need to check the parallel fifths between adjacent voices. In both cases, you need to check the bass for parallel fifths but not octaves.

Josephus. It seems like this system might take the artistic beauty out of music.

Aloysius. Josephus, my system is mainly useful for beginners, who need some affirmative guidance about what to do. Rather than negative prohibitions, it provides positive goals to shoot for—a kind of approximate and simplified idea of what their harmonizations should look like. Basically, close position is the basic configuration, with open the second most common

alternative. When you find yourself in the half-open or unusual configurations you should be expecting to “snap back” to one of these two basic positions relatively quickly. This gives you a way to make sensible choices from among the myriad options at your disposal!

Ambitious students don't need to be constricted by these techniques—once you master them. You should feel free to let your imagination roam. In particular, you should remember that the goal of counterpoint is to create *simultaneous melodies* that are all beautiful. So if you can think of an unusual configuration not described above, or an elegant way to get from the doubled interval to the half-open position, by all means use it! What I'm providing are approximations, rules-of-thumb about what *typically* happens.

The surprising thing is that composers like Bach managed to write great counterpoint even while making very heavy use of these rules of thumb. Fully 95% of the triadic voicing-leadingings in the chorales connect the five basic positions we've been discussing, and fully 85% of them conform to the preceding graph. My approximate theory may not give the last word on four-voice counterpoint, but it certainly provides a decent starting point.

Josephus. So are there any exercises that will help me internalize these ideas—not that I have any extra time, mind you, given all the homework you like to hand out?

Aloysius. Well, you might look through the Bach chorales, labeling triads with the letters O, U, C, and H. See for yourself how common the C configuration is, and how infrequently H progresses to H.

You can also harmonize using only using the C configuration, or the O configuration, or O, C, and H. Gradually add in the two main U configurations, making your melodic lines more sophisticated in the process.

You might also find it amusing to consider how to incorporate seventh chords into the system. Is there something like OUCH for seventh chords? Perhaps we could publish another dialogue discussing it!

Josephus. You know, my head is starting to feel a little better now. (*reaches down to start texting on his cellphone, but quickly thinks the better of it.*) Well, thank you exalted master ... and thanks for the chocolate.

Aloysius. See you later!

(*two minutes pass*)

Josephus (sticking his head back in the room). Hey, what about voice crossings? You didn't say anything about them! Doesn't the alto occasionally cross below the tenor?

Aloysius. Well, first of all, voice crossings, though reasonably common in the early 15th century, get progressively rarer over time. By the 18th century, they're almost extinct—with just 2% of Bach's triadic voicings containing crossings.

But it is possible to use crossings to embellish our basic schemas. For instance, examine the opening of Palestrina's “Adoramus, Te.” (The lowest staff here illustrates the basic voice-leading patterns.) The first five voice leadings are all based on the “3+1” trick—embellished, in two cases, with a crossing. (Without that crossing, there would be parallel fifths and octaves!)

The last two move from a close-position complete triad (G-C-E) to doubled interval (A-A-D) to a close-position complete triad (G \sharp -B-E).

the 3+1 trick

crossing doubled interval

I find it useful to think of crossings as embellishing our schemas in basically this way.

APPENDIX

Here is the percentage of triads by upper-voice configuration:

configuration		% of total
close		49
open		19
half-open		18
unusual	<i>octave plus</i>	6
	<i>doubled interval</i>	5
	<i>other</i>	3

The “other” subcategory includes “crossed” voicings (usually with alto below tenor, 2% of total) and a miscellany of odd voicings such as E3-G4-C5 (a close voicing with its bottom note down an octave).

Here’s a transition matrix showing the percentage of voice leadings from one configuration to another. Each number represents the percent of the total voice leadings that move from the row-label to the column-label. I’ve used boldface to identify the most likely destination for each type of voicing.

	doubled interval	close	half-open	open	octave plus
doubled interval	1	3	1	0	0
close	3	30	10	3	1
half-open	1	9	3	4	1
open	0	3	4	9	3
octave plus	0	1	1	3	1

The half-open→half-open category is probably artificially inflated by the fact that I am counting every triad as a genuine sonority. A human interpreter would likely say that some of these chords are “nonharmonic” as we will discuss later.